Mathematical Statistics Qualifying Exam

A 4-hour written exam, closed book, closed notes, no computer, no calculator.

Detailed Syllabus:

1. Basic probability theory
    a. Set theory, Kolmogorov axioms, Probability functions, and their properties, counting and enumerating outcomes
    b. Conditional probability, independent events
2. Univariate random variables
    a. Random variables, distribution functions, density functions, and probability mass functions
    b. Distribution of a transformed random variable
    c. Expected values, variance, moments, moment generating functions
    d. Common families of discrete and continuous distributions
    e. Exponential families, location and scale families
3. Multivariate random variables
    a. Joint, marginal, and conditional distributions
    b. Independent random variables
    c. Distribution of a multivariate transformation of random variables
    d. Hierarchical models and mixture distributions
    e. Multivariate normal distribution and the multinomial distribution
4. Random samples, inequalities, equalities, and convergence
    a. Random samples, sums of a random sample
    b. Sampling from normal distributions and derived distributions (t, F, and chi-square)
    c. Order statistics
    d. Chebychev's Inequality, Holder's inequality, Jensen's inequality
    e. Convergence in probability, convergence almost surely, and convergence in distribution
    f. Weak law of large numbers, central limit theorem, delta method
5. Statistical inference
    a. Sufficiency and likelihood principles
    b. Sufficient, minimal sufficient, ancillary, and complete statistics
    c. Factorization theorem
    d. Sufficient statistics for exponential families
6. Point Estimation
    a. Method of moments
    b. Maximum likelihood, the transformation property of the MLE
    c. Bayes Estimators
    d. Comparing statistical procedures, Risk function, Inadmissibility and admissibility, Mean squared error
    e. Properties of Estimators, Unbiasedness, Consistency, Mean-squared error consistency

    f. Search for the MVUE or the best unbiased estimator, Rao-Blackwell Theorem, Completeness, Lehmann-Scheffe, Cramer-Rao lower bound
7. Hypothesis testing
    a. Hypotheses, hypothesis testing procedures, critical region, types of errors, power function, level and size of a test, p-values
    b. Likelihood ratio tests (LRT), Bayesian tests, composite tests
    c. 0-1 loss function, risk function
    d. Most powerful tests, Neyman-Pearson Theorem
    e. Uniformly most powerful tests, monotone likelihood ratio
8. Interval estimation
    a. Inverting a test statistic, pivotal quantities, Bayesian intervals
    b. Size and coverage probability
    c. Test-related, Bayesian, and loss function Optimality
9. Asymptotics for statistical inference
    a. Consistency and efficiency
    b. Asymptotic distributions of LRT and other large-sample tests
    c. Approximate confidence intervals

Materials are from STAT 445 and STAT 446, and can be found in:
Statistical Inference by George Casella and Roger L. Berger, 2nd Edition, Chapters 1-10.
Sections that can be skipped:
6.4 (The Equivariance Principle)
7.2.4 (EM algorithm)
10.2 (Robustness)