

February 23, 2007

## **ABSTRACT ALGEBRA SYLLABUS**

### 1. Groups.

- A. Basic concepts: semigroups, monoids and groups; cyclic groups; cosets and Lagrange's Theorem; normality, quotient groups, and homomorphisms; symmetric, alternating, and dihedral groups; direct products and direct sums; free groups, free products, generators and relations.
- B. The structure of groups: free abelian groups; finitely generated abelian groups; group actions on a set; Sylow Theorems; nilpotent and solvable groups; Jordan-Hölder Theorem.

### 2. Rings and modules.

- A. Rings: rings and homomorphisms; subrings and ideals; integral domains; principal ideal domains and Euclidean domains; factorization in commutative rings; rings of quotients; rings of polynomials; factorization in polynomial rings.
- B. Modules: modules and homomorphisms; direct products and sums of modules; free modules; projective and injective modules; tensor products and *Hom* of modules; modules over a principal ideal domain.

### 3. Linear algebra

Matrices and linear maps; rank of a matrix; determinants; decomposition of a single linear transformation (including various canonical forms); characteristic polynomial, eigenvectors and eigenvalues; bilinear forms and dual spaces; inner product spaces; spectral theorem for normal matrices.

### 4. Fields and Galois Theory.

Field extensions; fundamental theorem of Galois theory; splitting fields; algebraic closure and normality; Galois group of a polynomial; finite fields; separability; cyclic extensions; solvability by radicals.

#### **References:**

- T. W. Hungerford, Algebra (Chapters I – V, VII and X)
- G. Birkhoff and S. MacLane, Algebra (All chapters except chapters XIV and XVI)
- K. M. Hoffman and R. Kunze, Linear Algebra (for item 3.)