Qualifying Exam Syllabus for Mathematical Neuroscience
Department of Mathematics
Case Western Reserve University

1. Electrophysiology

A. Equilibrium descriptions of cell membranes. Nernst potential. Goldman-Hodgkin-Katz equations.

B. Hodgkin-Huxley equations. Action potential initiation. Action potential propagation (unmyelinated axon).

C. Ion channel models (ordinary differential equation, stochastic differential equation, Markov process descriptions). Types of ion channels.

2. Reduced models, excitability and bifurcation structure

A. SNIC bifurcations, type I neurons, and the Ermentrout/Koppel theta model. Hopf bifurcations (sub/supercritical), type II neurons and phase models. Fitzhugh-Nagumo, and Morris-Lecar models. Izhikevich's minimal models taxonomy.

B. Noisy integrate-and-fire models (ODE, SDE, Fokker-Planck descriptions; first passage times and interspike interval distributions).

C. Action potential propagation and traveling wave solutions in reduced models (e.g. Fitzhugh-Nagumo in a 1D medium).

D. Geometry of square wave and elliptic bursting.

3. Small Networks

A. Synapses: fast and slow, excitatory and inhibitory. Neurotransmitter types, functions and mechanisms.

B. Local and global stability analysis for a pair of coupled oscillators. Entrainment of reduced models driven by periodic and aperiodic stimuli.

C. Central pattern generator circuits: examples.

4. Large Networks

A. Wilson-Cowan equations.

B. Pattern formation in mean field network models.

C. Synchronization phenomena in mean field network models, and applications (binding hypothesis; models of epilepsy).

5. Synaptic Plasticity

A. Hebb's rule. BCM rule, correlation and anti-correlation based learning models. Models for spike time dependent plasticity.

B. Perceptron; feedforward classification networks; backpropagation learning algorithm.

6. Computational Methods

A. The student should have familiarity with implementing models of each of the preceding categories in at least one simulation environment such as NEURON, GENESIS, XPP, MATLAB, SNAAP, or similar.

References

Primary references:

Dynamical Systems in Neuroscience (Izhikevich, MIT Press 2007)
Neural networks as spatio-temporal pattern-forming systems (Ermentrout 1998), *Reports on progress in physics*.

Secondary references:

The NEURON Book (Carnevale & Hines)
Simulating, Analyzing, and Animating Dynamical Systems, a guide to XPPAUT (Ermentrout)
Spikes, Decisions and Actions (Wilson)
Theoretical Neuroscience (Dayan & Abbott)
Methods in Neuronal Modeling (Koch & Segev, editors)
Biophysics of Computation (Koch)
Elements of Applied Bifurcation Theory (Kuznetsov)
Handbook of Stochastic Methods (Gardiner)

The student may also find many valuable references in mathematical and computational neuroscience in the annals of scholarpedia, the online peer-reviewed encyclopedia curated by E. Izhikevich.
http://www.scholarpedia.org/article/Encyclopedia_of_computational_neuroscience

---
Last updated *June 11, 2009*