

May 7, 2007

Geometry Syllabus

I. DIFFERENTIAL GEOMETRY

1. Geometry of Curves. Parametrizations, arc length, curvature, torsion, Frenet equations, global properties of curves in the plane.
2. Extrinsic Geometry of Surfaces. Parametrizations, tangent plane, differentials, first and second fundamental forms, curves in surfaces, normal and geodesic curvature of curves.
3. Intrinsic and Extrinsic Geometry of Surfaces. Frames and frame fields, covariant derivatives and connections, Riemannian metric, Gaussian curvature, Fundamental Forms and the equations of Gauss and Codazzi-Mainardi.
4. Geometry of geodesics. Exponential map, geodesic polar coordinates, Fermi coordinates along a curve. Properties of geodesics, Jacobi fields, convex neighborhoods.
5. Global results about surfaces. The Gauss-Bonnet Theorem, Hopf-Rinow theorem, cut points and conjugate points, the Bonnet-Myers theorem.

References:

- M. Do Carmo, Differential Geometry of Curves and Surfaces.
B. O'Neill, Elementary Differential Geometry.
D. Struik, Lectures on Classical Differential Geometry.
J. A. Thorpe, Elementary Topics in Differential Geometry.

II. DIFFERENTIABLE MANIFOLDS

1. Foundational material. Definition of smooth manifold, tangent bundle, vector field, smooth maps, differentials, induced maps.
2. Examples of manifolds. Inverse and implicit function theorems, submanifolds and varieties, Lie groups.
3. Vector bundles and vector fields. Picard's Theorem (solutions of ODE's), tensors and tensor calculus, differential forms and exterior derivative.
4. Lagrangian and Hamiltonian formulation of mechanical systems on manifolds, Legendre transformations, conservation laws, Riemannian metrics.

References:

- V. Arnold, Mathematical Methods of Classical Mechanics, Chapters 1-4 and 7.
L. Auslander and R. MacKenzie, Introduction to Differentiable Manifolds.
M. Spivak, Comprehensive Introduction to Differential Geometry, Volume 1.

Note: The above syllabus is based on the contents of MATH 465 and MATH 467 and should be construed as a sample syllabus. Should this exam be offered in the future, the selection of topics may be modified, for example depending on the coursework of a particular student, or to ensure the breadth and non-overlap requirements.