May 7, 2007

<u>Requirements: Qualifying Exam in Functional Analysis</u>

  - Supporting hyperplanes, extreme points
  - Properties of convex sets
  - Krein-Milman theorem, exposed points
- Stone Weierstrass theorem
- Normed vector spaces. Banach spaces.
  - Linear Functionals. Dual spaces. Hahn-Banach theorem and consequences: Separation theorems (in finite and infinite dimensional setting).
  - Baire Category theorem. Open Mapping theorem. Closed Graph theorem. Uniform Boundedness Principle.
  - Topological Vector spaces. Weak-and weak* -topologies. Banach-Alaoglu theorem.
  - L_p-spaces and their duals.
  - Hilbert spaces. Orthogonality. Riesz Representation theorem. Orthonormal sets of vectors and bases. Isomorphic Hilbert spaces and Fourier transform for the circle. Direct sum of Hilbert spaces,
  - Positive Linear functionals on $C_c(X)$. The Dual of $C_0(X)$.
  - Operators on Hilbert space. Adjoint of an operator. Projections. Invariant and reducing subspaces. Compact operators. Diagonalization of compact self-adjoint operators.
  - Wave equation. Heat equation
  - Fourier series: Uniqueness. Convergence. Convolutions

<u>References</u>:

G. B. Folland: <u>Real Analysis</u>. Sections 5.1-5.5, 6.1-6.3 and 7.1-7.3

J. B. Conway: <u>A Course in Functional Analysis</u>. Chapters I.1-II.5

L. D. Berkovitz: <u>Convexity and Optimization in</u> R^n. Chapters I and II.

E. Stein and R. Shakarchi: <u>Fourier Analysis</u>. Chapters 1, 2 and 3.1

Note: The above syllabus includes most of MATH 424 and MATH 427, as well as some other selected topics. It should be construed as a sample syllabus. Should this exam be offered in the future, the selection of topics will depend on particular circumstances of the student(s) in question (such as, for example, the coursework taken by the student, or to ensure the breadth and non-overlap requirements). In particular, should the students attempt also *Real Analysis*, the topics coming from MATH 424 will be replaced by additional topics to limit the overlap between the two exams.