February 25, 2007

TOPOLOGY SYLLABUS

I. *GENERAL TOPOLOGY*

1. Set theory. Orderings. Cardinals and ordinals. Equivalence of the various forms of Zorns's lemma. Transfinite induction and construction.

2. Topological spaces. Neighborhood system of a point; closure, interior and boundary; bases and subbases. Connectedness, arcwise connectedness, local connectedness.

3. Moore-Smith convergences and filters. Directed sets and nets, subnets and cluster points. Limits and iterated limits, sequences and subsequences.

4. Product and Quotient spaces. Continuity of maps, homeomorphisms. Open maps and closed maps.

5. Embedding and Metrization. Existence of continuous functions, Tietze's theorem. Embedding in cubes. Metric spaces. Completeness. Baire category theorem. Urysohn metrization theorem.

6. Compactness and local compactness. Tychonoff's theorem. Compactification (Alexandroff one-point and Stone-Cech). Lebesgue's covering lemma. Paracompactness.

References:

J. L. Kelley, General Topology, Chapters 1-3; Chapter 4, up to and including Theorem 17; Chapter 5.

J. Dugundji, Topology, Chapters I-VII, IX (Sections 1-8), X, XI, XIV (Sections 1-7).

J. Munkres, Topology, a first course, Chapters 1-5.

II. *ALGEBRAIC TOPOLOGY*

1. Homotopy theory. The fundamental group, functorial properties, Van Kampen's theorem. Covering spaces: unique path lifting, the universal cover, relation between covering spaces and fundamental group. Computations with 2-manifolds. Higher homotopy theory, e.g. Brouwer's fixed point theorem.

2. Homology theory. Basic properties of homology groups. Chain complexes and homotopy. The Hurewicz theorem. Short and long exact sequences; the long exact sequence of a pair; the five-lemma; Mayer-Vietoris sequence. Homology of 2-manifolds and low-dimensional simplicial complexes. Applications: fixed point theorems; vector fields on spheres. Universal coefficients, Kunneth formula.

References:

E. H. Spanier, Algebraic Topology, Chapter 1; Chapter 2, Sections 1-6; Chapter 3, Sections 1-4 and 8; Chapter 4. Additionally, all exercises in Chapters 1-4, and look at Chapter 7, Sections 1, 2 and 5.

W. S. Massey, Algebraic Topology: an introduction, Chapters 1, 2, 3; Chapter 4, Sections 1-5; Chapter 5.

W. S. Massey, Singular Homology Theory, Chapters 1-3.

J. Munkres, Topology: a First Course, Chapters 8 (a good reference for π and covering spaces).

J. W. Vick, Homology Theory, Second Edition, Chapters 1-5