February 23, 2007

REAL ANALYSIS SYLLABUS

1. Real and complex number systems. Elements of point-set topology of Euclidean space. Numerical sequences and series. Continuity and differentiability for functions of one and several variables. Riemann-Stieltjes integral. Sequences and series of functions. Fourier series, Fejér's theorem. Inverse and implicit function theorems. Differential forms. Stokes theorem.

   Reference:

   W. Rudin, Principles of Mathematical Analysis, 3rd ed.

   Suggested additional reference (for the last three topics):

   M. Spivak, Calculus on Manifolds

2. General theory of measure and integration. Measures and outer measures. Lebesgue measure on R^n. Integration. Convergence theorems. Product measures and Fubini's theorem. Signed measures. Hahn-Jordon decomposition, Radon-Nikodym theorem, Lebesgue decomposition. Differentiation and absolute continuity with respect to Lebesgue measure. L^p spaces. Measures on locally compact spaces, Riesz representation theorem (for functionals on $C_0(X)$).

3. Elements of functional analysis. Normed linear spaces. Hahn-Banach, Banach-Steinhaus, open mapping, closed graph theorems, with some applications in classical analysis. Weak topologies, Banach-Alaoglu theorem. Function spaces, Stone-Weierstrass and Arzelà-Ascoli theorems. Basic Hilbert space theory. L^2 theory of Fourier series.

   Primary Reference:

   G. B. Folland, Real Analysis, Wiley, 2nd ed.
   Chapters 1-3, 5, 6 (Sections 1, 2), 7 (Sections 1-3).

   Suggested additional references:

   H. L. Royden, Real Analysis, 2nd Edition, Chapters 1-14.
   W. Rudin, Real and Complex Analysis, 2nd Edition, Chapters 1-9.
   C. Goffman and G. Pedrick, First Course in Functional Analysis, Chapters 1-4.