Inhomogeneity effects on observations in cosmology:
SN observations and lensing of the CMB

George Ellis,
University of Cape Town

Sesto Workshop

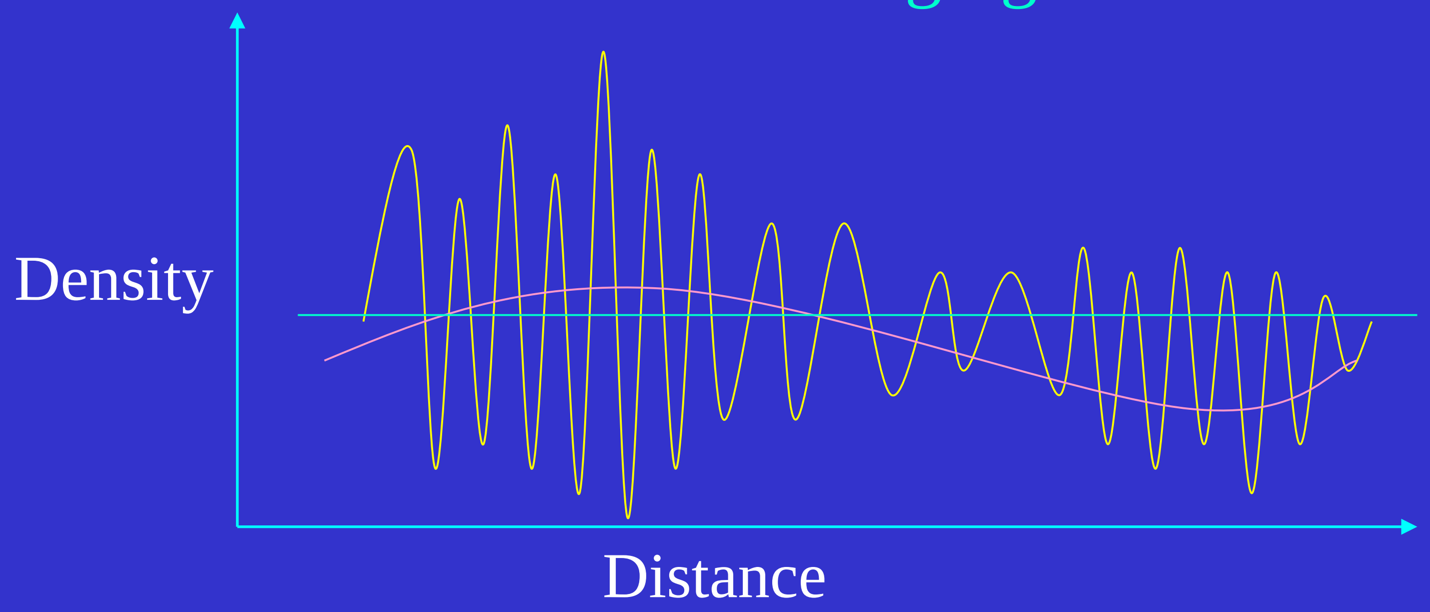
July 17, 2018

Local inhomogeneity: description

G.K. Batchelor: An Introduction to Fluid Dynamics, Cambridge University Press (1967).

Multiple scales of representation of ***same*** system

Different averaging scales



Stars, clusters, galaxies, universe

1. Viewing perturbations via maps of the background space into the lumpy universe, may be a fruitful way to go

2. As regards lensing of the Cosmic Microwave Background

   (a) distance measures and their relation to affine parameters,

   (b) the limits of the integrals involved,

   (c) the `wrinkly surface' argument and Sach's shadow theorem.

3. The effect in both cases of cusps in the past light cone due to strong gravitational lensing

4. Selection effects are never luminosity limited

5. SN: the Ricci/Weyl tensor issue, bias arising from preferred (emptier) pencils of geodesics,

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PHYSICAL REVIEW D VOLUME 40, NUMBER 6 15 SEPTEMBER 1989

Covariant and gauge-invariant approach to cosmological density fluctuations

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(Received 4 April 1989)

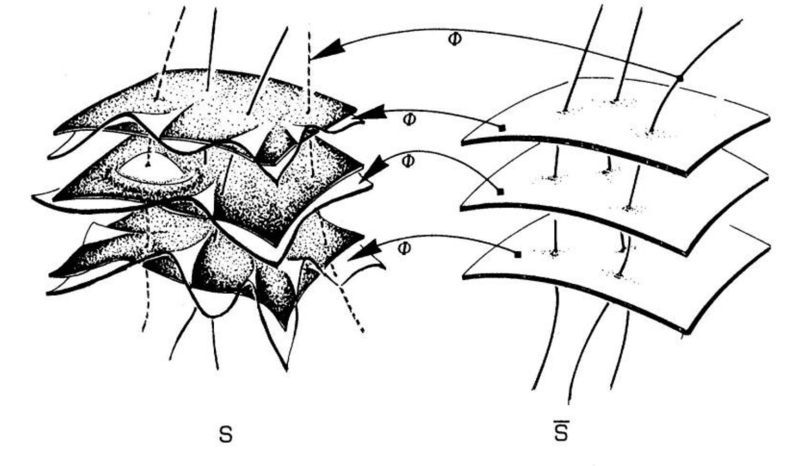

FIG. 1. The perturbed density $\delta\mu$ is defined by a mapping Φ of an idealized world model $\bar{S}$ into a more accurate world model S, for Φ maps surfaces $\{\bar{\mu} = \text{const}\}$ from $\bar{S}$ into S, where they can be compared with the actual surfaces $\{\mu = \text{const}\}$.

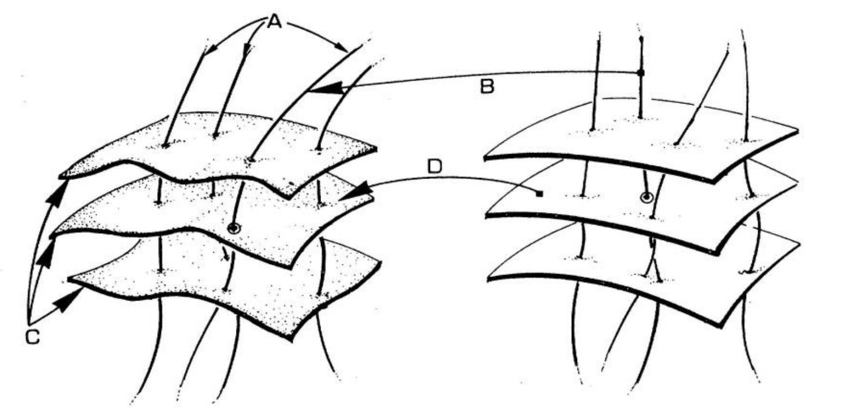

FIG. 2. The map Φ has four aspects: (A) choice of a family of time lines in each spacetime; (B) choice of a particular correspondence of time lines in the family in $\bar{S}$ to particular time lines in the family in S; (C) choice of a family of spacelike surfaces in each spacetime; (D) choice of a particular correspondence of surfaces from the family in $\bar{S}$ to surfaces in the family in S.

Use 1+3 covariant and gauge invariant variables in the real lumpy space; then approximate the full non-linear equations

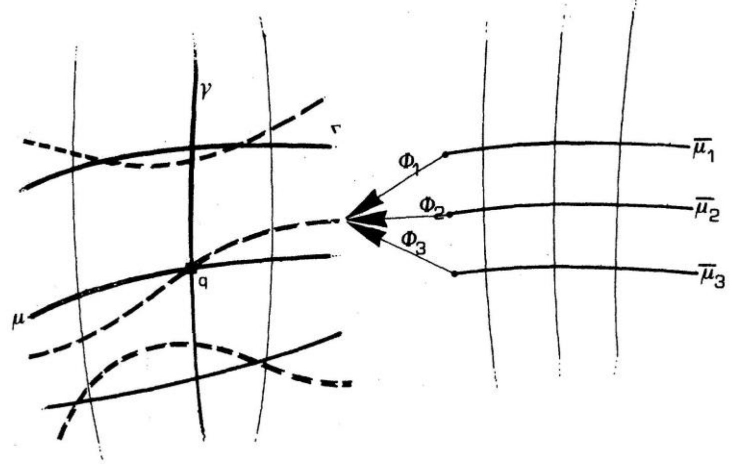

FIG. 3. By varying the assignation (D) of particular surfaces in $\bar{S}$ to surfaces in S, we can give the density perturbation $\delta\mu = \mu - \bar{\mu}$ at the event q in S (where the world line γ intersects the surface $\{\bar{\mu} = \text{const}\}$) any value we like.

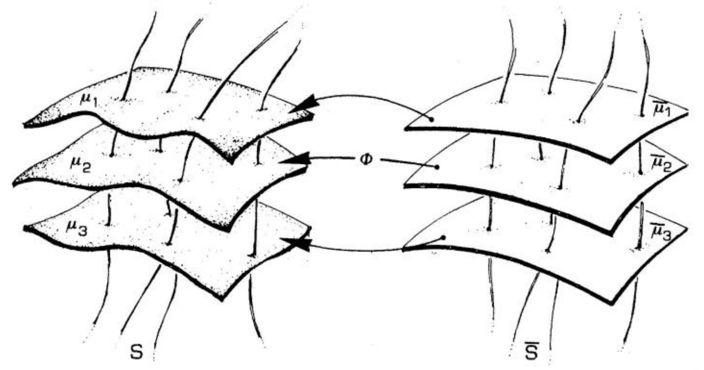

FIG. 4. By choosing Φ so that the surfaces $\{\bar{\mu} = \text{const}\}$ in S are the same as the surfaces $\{\mu = \text{const}\}$, and then choosing the correspondence (D) to assign the same numerical values to $\bar{\mu}$ on each surface as μ has on it, we obtain a *zero density-perturbation* gauge. Note that the proper time τ between any two of these surfaces in S will vary spatially, in general; the physical density variation is coded in this spatial variation of $dt/d\tau$.

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2. Lensing of the Cosmic Microwave Background

Clarkson, C., Umeh, O., Maartens, R., and Durrer, R. (2014). "What is the distance to the CMB?". *Journal of Cosmology and Astroparticle Physics* **11**: 036 https://arxiv.org/abs/1405.7860

Kaiser, N, and Peacock, J A (2015). "On the bias of the distance-redshift relation from gravitational lensing." *Monthly Notices of the Royal Astronomical Society* **455**: 4518-4547 https://arxiv.org/abs/1503.08506

Bonvin, C , Clarkson, C, Durrer, R, Maartens, and Umeh, O (2015) "Do we care about the distance to the CMB? Clarifying the impact of second-order lensing" *JCAP* **2015**: 050 https://arxiv.org/abs/1503.07831

Note on the Kaiser-Peacock paper regarding gravitational lensing effects

arXiv preprint arXiv:1806.09530

George F R Ellis · Ruth Durrer

Redshift

2.1 Redshift

For a family of observers with future directed 4-velocity u^a $(u^b u_b = -1)$ one finds the basic redshift formula

$$\boxed{1 + z = \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{(u_a k^a)_{\text{emitter}}}{(u_b k^b)_{\text{observer}}}\,.} \tag{1}$$

We consider the past-directed null geodesic curve $x^a(v)$, where v is an affine parameter, with tangent vector k^a pointing from the observer to the source:

$$\boxed{k^a = \frac{dx^a}{dv} : \; k^a k_a = 0, \;\; k^a{}_{;b} k^b = 0.} \tag{2}$$

If the parameter v is not affine, one will get instead

$$k^a k_a = 0, \;\; k^a{}_{;b} k^b = f(v) k^a \tag{3}$$

for some function $f(v) \neq 0$. We note here that if $k_a = \phi_{;a}$ for a scalar field ϕ, it is automatically affine Thus this will indeed be true for the tangent vector field to the past null cone $w = const$, because then $k_a = w_{,a}$ But it will not automatically be true if one calculates along individual null geodesics or bundles of null geodesics with some geometrically chosen curve parameter v such as cosmic time t or distance travelled d.

The direction n^a of a past directed light ray relative to a (future directed) 4-velocity u^a is

$$k^a = (-u^a + n^a)(u^b k_b), \;\; n^a n_a = 1, \;\; n^a u_a = 0. \tag{4}$$

Redshift

and

$$\boxed{dl = (u^a k_a)dv} \tag{5}$$

is the projected spatial distance along the direction n^a of the null vector corresponding to a curve parameter increment dv.

The change of observed wavelength λ down a past null geodesic for more and more distant sources is given by the derivative of (1) down the past null cone relative to an affine parameter and using the formula

$$u_{a;b} = \theta_{ab} + \omega_{ab} - \dot{u}_a u_b \tag{6}$$

to give

$$\boxed{\frac{d\lambda}{\lambda} = \{\theta_{ab} n^a n^b + (\dot{u}_a n^a)\} dl} \tag{7}$$

where θ_{ab} is the fluid expansion tensor, combining an isotropic expansion term θ and a shear term σ_{ab}, and $\dot{u}_a$ is the fluid acceleration The first term is the Doppler shift term, with isotropic and anisotropic parts, and the second the gravitational redshift term (zero for a geodesic congruence). If the curve parameter is not affine, relation (7) will have an extra term: the affine condition is assumed in its derivation,

Temperature

2.2 Temperature

The surface intensity I_ν of radiation received from a source with spectrum $\mathcal{I}(\nu)$ and surface brightness $I_G(n^a)$ in the direction n^a is given by

$$\boxed{I_\nu(n^a)d\nu = I_G(n^a)\frac{\mathcal{I}(\nu)(1+z)d\nu}{(1+z)^3}} \tag{8}$$

where the redshift is given by (1). This relation follows from reciprocity theorem which is generically true. We can parametrise the direction n^a as usual by spherical angles (θ, ϕ). Applying (8) to black body radiation, it follows that in each direction (θ, ϕ), radiation emitted as black body radiation at temperature $T_{\text{emit}}(\theta, \phi)$ at the point on the last scattering surface (LSS) in that direction is received as black body radiation at temperature $T_{\text{obs}}(\theta, \phi)$ where

$$\boxed{T_{\text{obs}}(\theta, \phi) = \frac{T_{\text{emit}}(\theta, \phi)}{(1+z)}.} \tag{9}$$

Here (θ, ϕ) denotes the direction of n^μ at the observer which equals the one at the emitter e.g. in geodesic light cone coordinates This remarkable relation, resulting from a combination of results from quantum theory, statistical mechanics, and general relativity, is an exact relation true in any cosmological spacetime.

Decoupling

2.3 Decoupling

Decoupling of the CMB from matter takes place when the baryon temperature T_m drops below the ionisation temperature T_{dec} so that negatively charged free electrons in the primordial plasma combine with positively charged nuclei to become neutral hydrogen and Thomson scattering ceases. Because the baryons and radiation are in equilibrium to a good approximation at that time, their temperatures are equal:

$$T_\gamma = T_m. \tag{10}$$

Decoupling therefore happens when the radiation temperature drops below T_{dec}; that is , when

$$\boxed{T_\gamma = T_{\text{dec}}.} \tag{11}$$

Hence setting $T_{\text{emit}}(\theta, \phi) = T_{\text{dec}}$ in (9) the observed CMB temperature is

$$\boxed{T_{\text{obs}}(\theta, \phi) = \frac{T_{\text{dec}}}{(1+z)}.} \tag{12}$$

Decoupling

Now T_{dec} is fixed by the physics of recombination of hydrogen (we ignore the issue of the ionisation of helium) and $T_{\text{obs}}(\theta,\phi)$ is the measured CMB temperature, which varies over the sky. So what the measured CMB temperature in any direction tells us is the redshift $z_*(\theta,\phi)$ of the emission surface (the 'cosmic photosphere') in that direction:

$$\boxed{(1+z_*(\theta,\phi)) = \left(\frac{T_{\text{dec}}}{T_{\text{obs}}(\theta,\phi)}\right)} \tag{13}$$

where the numerator is given by the physics of ionisation and the denominator is the observed CMB temperature. As remarked by Sachs and Wolfe the interpretation of this redshift as being due to Doppler or gravitational effects (e.g. whether it is due to the Rees-Sciama effect or local redshifts relative to the cosmological expansion) does not affect this formula, which is completely general. The conclusion is

Lemma 1 *Decoupling of matter and radiation ('the cosmic photosphere') takes place at the redshift $z = z_*(\theta,\phi)$ determined in terms of $T_{obs}(\theta,\phi)$ by (13), irrespective of the causes of that redshift. Hence the correct boundary condition to use in evaluating lensing effects on the CMB is to determine the cosmic photosphere by setting $z = z_*(\theta,\phi)$ on the null geodesic in direction (θ,ϕ) for all θ,ϕ.*

The COBE, WMAP, and Planck images of the CMB are therefore just images of the variation of z_* across the sky.

(a) distance measures and their relation to affine parameters,

3 Curve parameter

As pointed out above, the standard formula (7) assumes that the null geodesic is affinely parametrised. A key issue then in doing CMB calculations is whether an affine parameter is used along the relevant geodesics, or some other parameter. Equivalently, what distance measure is chosen in the calculations? If it is for example chosen as comoving distance, however that is defined, relation (7) may no longer hold.

Eqn (1) in KP is the null geodesic focussing equation, which uses an affine parameter, as does the geodesic deviation equation. However the caption to figure 1 says, "*In a hypothetical universe with inhomogeneity in some finite region of space, consider the mean fractional change to the area of a surface of constant redshift, or cosmic time*", which they compare with a surface of constant distance travelled. Now a surface of constant time in a perturbed FLRW model is a gauge dependent quantity (it depends on the time parameter chosen in the inhomogeneous model), and there is no reason why it should

(a) distance measures and their relation to affine parameters,

be a surface of constant redshift, so these are generically two different surfaces. KP also refers to "the radius reached by the light rays" and a "a path length λ", which are presumably both the integral of (5) down the null geodesic. A variety of different distance measures are being used (they may be the same in the background model, but will not be so in the perturbed model).

However KP then state

> **KP Claim 1** : *The rest of the paper consists of a calculation of the perturbation to the area of a surface of constant redshift.*

As long as this is what is actually done, and this is confirmed at the start of Section 4.1 in KP, the confusion about what curve parameter is used need not matter: the cosmic photosphere is being treated as a surface of constant redshift. The perturbation to the area of that surface, with consequent changes in apparent distance, is due to gravitational lensing effects (the focussing equation and perhaps time delay effects) as discussed in depth by KP. However if either the null focussing equation or the geodesic deviation equation is used to deduce area changes, then it does matter if the curve parameter is affine or not; and similarly if (7) or an equivalent equation is used to deduce the change of redshift down a family of null geodesics, then it matters as well.

(b) the limits of the integrals involved,

4 Curve end-point

Following Weinberg, the curve endpoint in KP is taken (see KP Claim 1) as being on a surface of constant redshift But Section 2.3 above implies this cannot be correct when examining CMB lensing, if we use the correct physical conditions for decoupling.

Corrollary 2 *Equation (12) shows that if one calculates the CMB temperature $T_{calc}(\theta, \phi)$ using as endpoint a surface of constant redshift z_{const} with correct physical conditions for decoupling, then $T_{calc}(\theta, \phi)$ will have no angular variation whatever:*

$$T_{\rm calc}(\theta, \phi) = \frac{T_{\rm dec}}{(1 + z_{\rm const})} \tag{17}$$

where the RHS is constant.

Hence taken at face value, the KP calculation does not give what is needed for CMB calculations in a perturbed FLRW model, where the observed CMB temperature varies due to varying redshifts of emission in different directions of observation in the sky.

Corollorary 3 *The observed CMB anisotropy $T_{obs}(\theta, \phi)$ is due to the difference in redshift in the inhomogeneous universe between a chosen reference such as a surface of constant redshift $z = z_{const}$ and the physical surface of decoupling $z = z_*(\theta, \phi)$ given by (13).*

(b) the limits of the integrals involved,

This difference for example determines all the anisotropies detected by the Planck satellite observations. The issue is in fact acknowledged in Appendix A2 of KP, but they do not explain how they resolve it. But that is the heart of the physical effect.

This approach is satistactory in the linear case, but becomes very opaque in the non-linear case when one mixes a variety of distance measures as KP do (in A2: redshift, in A3 and A4: distance along the light ray, in A5: optical path length and conformal path length, finally in A6: redshift, as per KP Claim 1). This approach contrasts with the view proposed here where, one works as far as possible in the real inhomogeneous universe, and uses the physics of decoupling to determine the integration endpoint.

(b) the limits of the integrals involved,

That analysis depends on temperatures rather than densities. How does this relate to surfaces of constant density? For radiation,

$$\rho_\gamma = aT_\gamma^4 \;\Rightarrow\; \delta\rho_\gamma = 4aT_\gamma^3\delta T_\gamma \tag{14}$$

and pure adiabatic perturbations are characterised by

$$\frac{\delta\rho_\gamma}{\rho_\gamma} = \frac{4}{3}\frac{\delta\rho_m}{\rho_m} \;\Leftrightarrow\; \delta\rho_m = 3\rho_m\frac{\delta T_\gamma}{T_\gamma}. \tag{15}$$

Consequently in the adiabatic case,

$$\{\delta T_\gamma = 0\} \Rightarrow \{\delta\rho_m = 0\}. \tag{16}$$

Corollary 1 *In the adiabatic case, by (11) and (16) the LSS is a surface of constant baryon density* $\delta\rho_m = 0$*, so in that case the observed CMB fluctuations do not represent density fluctuations, as is often stated.*

In standard perturbation theory language, this shows that in uniform density gauge (which for adiabatic perturbation is the same as uniform temperature gauge) the density fluctuations are given exactly by the redshift fluctuations. In the non-adiabatic case this will no longer be true.

(c) the `wrinkly surface' argument and Sach's shadow theorem

5 The 'wrinkly surface' argument

KP state that time delays cause a further effect: "*the surface is 'wrinkled' owing to time delays induced by the density fluctuations ... one can draw an analogy with the surface of a swimming pool perturbed by random waves of small amplitude. These cause a fractional increase in the area of the surface that is on the order of the mean square tilt of the surface*", which they call the 'wrinkly surface' argument.

However one must take into account Sach's shadow theorem which states that the shape and area of an image in a screen orthogonal to the light ray are independent of the velocity of an observer. To show this, consider a vector x^a lying in a screen orthogonal to k^a, which is effectively what the LSS is for the observer; then

$$x^a k_a = 0. \tag{18}$$

The screen is set perpendicular to the incoming light ray, else there will be projection effects simply due to the screen being at an angle relative to the direction of observation, as opposed to any effects caused by time delays, which are equivalent to different distances down the light cone. Then changing distance down the null cone by a parameter distance dv (that is, a time delay effect) adds a null increment

$$\boxed{dx^a = k^a dv, \;\; k_a k^a = 0,} \tag{19}$$

to each such vector x^a, equivalent to a sum of a time displacement and a spatial displacement. The increment (19) does not alter condition (18):

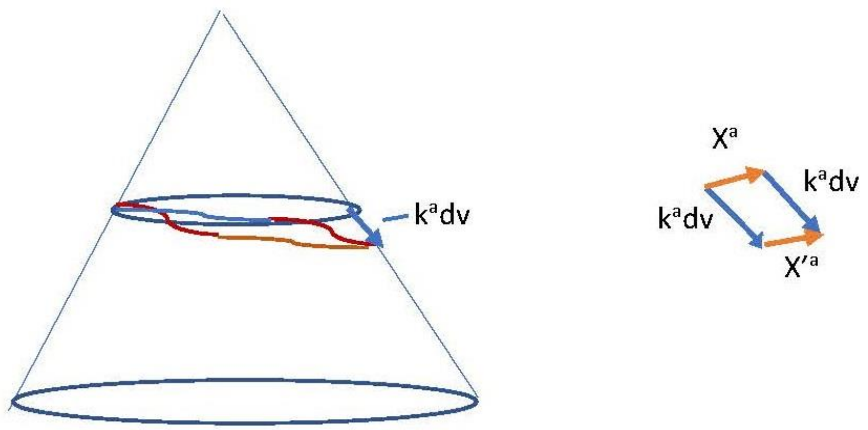
$$\{(x')^a = x^a + dv\, k^a, \;\; x^a k_a = 0\} \;\Rightarrow\; \{(x')^a k_a = 0\}. \tag{20}$$

(c) the `wrinkly surface' argument and Sach's shadow theorem

Sachs pointed out that such an increment does not change any magnitude or shape in the image: by (19) applied to x^a and a similar screen vector y^a,

$$\boxed{\{(x')^a = x^a + dv\, k^a,\ (y')^a = y^a + dv\, k^a\} \Rightarrow \{(x')^a (y')_a = x^a y_a\}.} \tag{21}$$

In particular, $(x')^a (x')_a = x^a x_a$. This applies to the CMB case if we regard the LSS as the screen space relative to our observations.



One can always choose conformally flat coordinates, in the lumpy spacetime or the background

(c) the `wrinkly surface' argument and Sach's shadow theorem

Lemma 2 *Lengths, angles, and areas in a screen space are unchanged by altering that space by a small amount down the past light cone, adding an extra spatial displacement and time displacement that together represent a null displacement for each point in the screen space.*

On the view taken here, one compares the real decoupling surface in the inhomogeneous spacetime with the image in that spacetime of the decoupling surface in the background model. Then the shadow theorem applies down the real past light cone in the inhomogeneous spacetime, and there is no such effect.

Corollorary 4 *Applying Lemma 2 to the LSS in the real physical spacetime as compared to the image in that spacetime of the LSS in the background model, there is no wrinkly surface effect for the CMB emission surface.*

(c) the `wrinkly surface' argument and Sach's shadow theorem

KP by contrast consider the issue in the background spacetime and find a non-zero result; but the shadow theorem also applies there, in regard to the background lightcone. However the past light cone in the background spacetime does not correspond to the image of the physical light cone in the inhomogeneous spacetime, ... : the imaged light cone is not a null surface. There can consequently be such an effect resulting from the mapping between the physical spacetime and the background spacetime, which is of course a gauge dependent relation. Working this out one must again show how the relation between the real surface of decoupling and a fictitious one works, which requires identifying physically the real surface of decoupling, which is not a surface of constant redshift (in contrast to KP Claim 1). It also requires taking fully into account the Minkowski geometry that leads to the Shadow Theorem; it is not clear that KP does this.

The view of this note is that it would be clearer to consider such effects in the physical spacetime, where it vanishes.

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3. Folds and caustics in past light cone

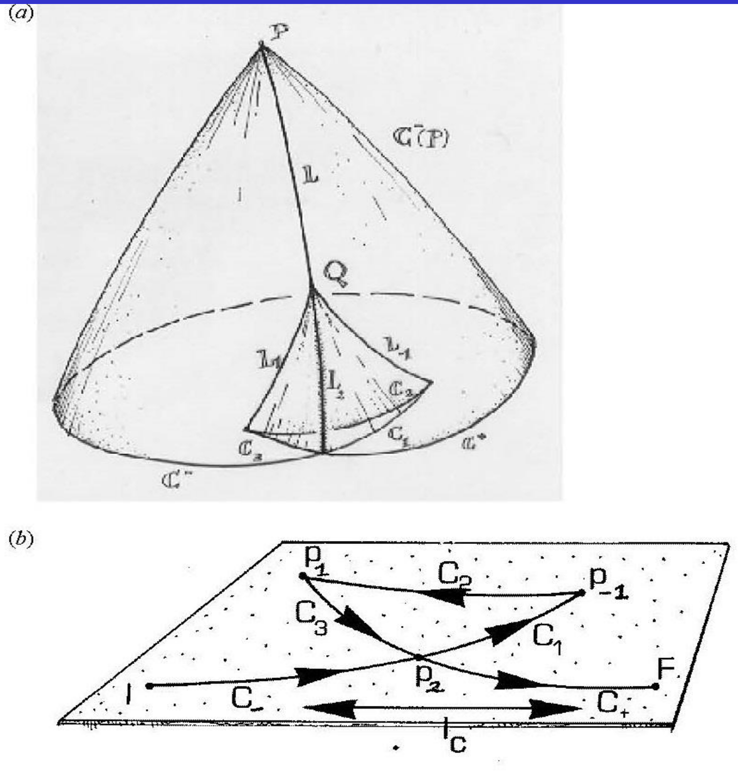


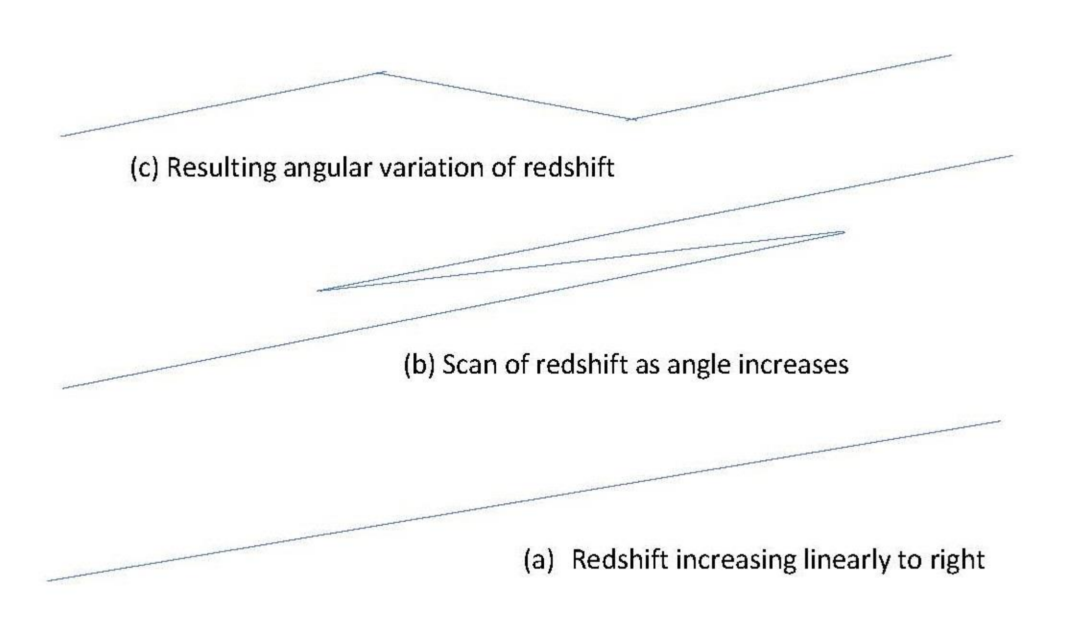
Figure 1. *(a)* A lens *L* and resulting caustics on the past lightcone $C^{-}(P)$ (two-dimensional section of the full lightcone), showing in particular the cross-over line L_2 and cusp lines L_{-1}, L_1 meeting at the conjugate point *Q*. The intersection of the past lightcone with a surface of constant time defines exterior segments C^{-}, C^{+} of the lightcone together with interior segments C_1, C_2, C_3. *(b)* The imaged point moves forward along C_1 from *I* to the cusp at P_{-1}, backward along C_2 to the cusp at P_1, and then forward along C_3 to *F*.

CQG 15: 2345 (1998)
Ellis, Bassett, Dunsby

Every strong lens causes a fold plus cusps

Real past light cone has billions of caustics, hierarchically structured

Single lens effect on CMB observations



Detect individual lensing masses this way via the CMB,
Then find their causes by multi-wavelength observations?

Issues: (a) Imaging PSF (b) Silk damping
But (c) NO issue of aligment. LSS covers whole sky!

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4. Source selection effects are never luminosity limited

HST, SDSS, Planck, SKA, Euclid, LSST, etc

- Fantastic technology

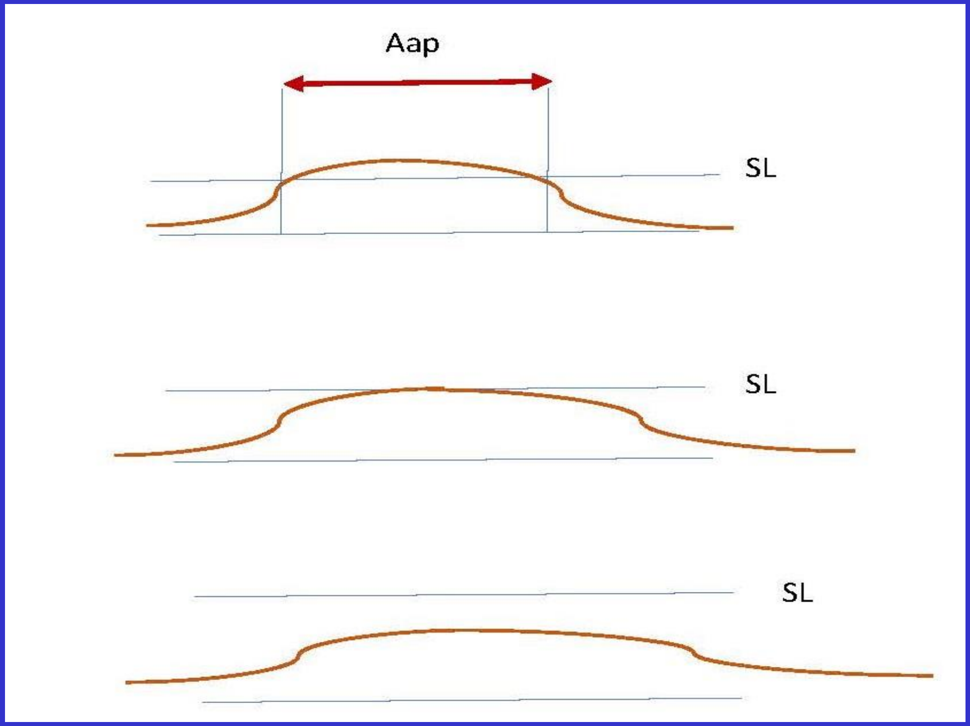
- Never flux/luminosity limits for sources:

Selection and detection depend on observed (a) surface brightness (b) angular size

$\Rightarrow$ Magnitude + Scale size + redshift/cosmology

Ellis Perry and Sievers *AJ* **89:** 1124 **(**1984)

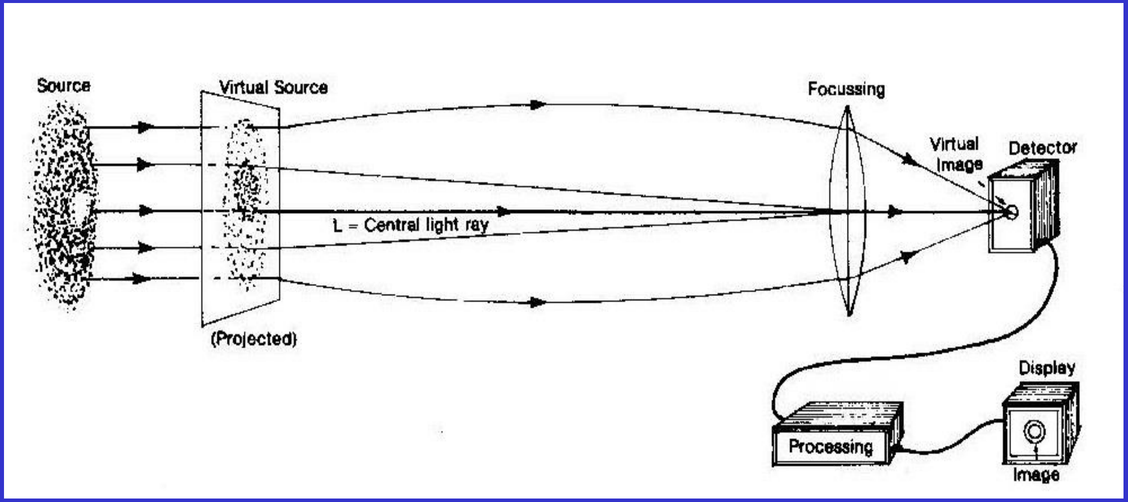
Low surface brightness galaxies (Disney: Nature 1976)

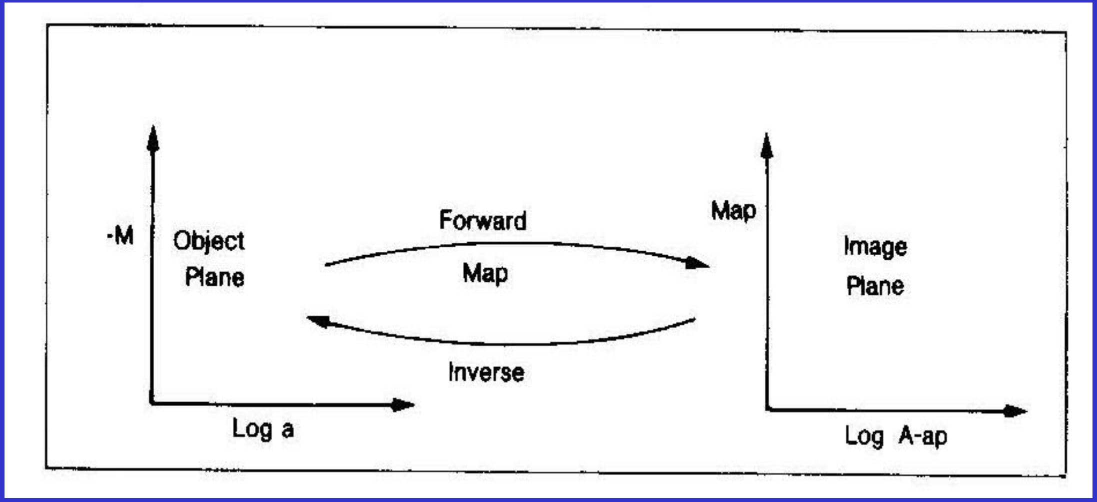


Increase radius a(t) of galaxy keeping luminosity L constant.

Apparent size drops to zero as central surface brightness drops below detection limit SL; galaxy becomes invisible

Never flux limits

- 



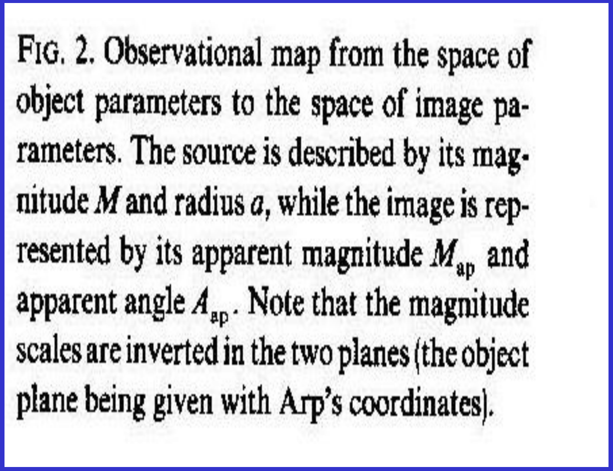
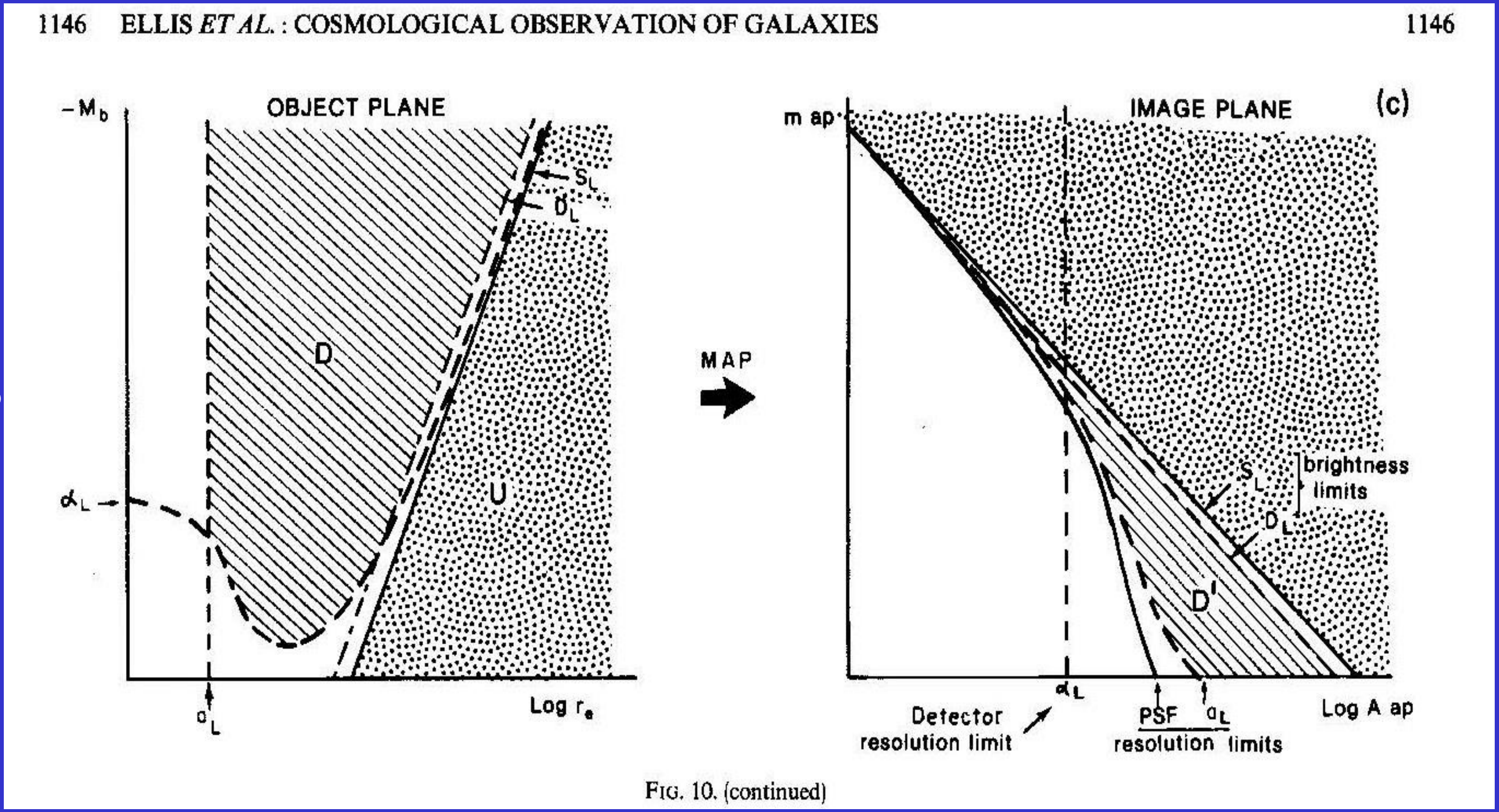


FIG. 2. Observational map from the space of object parameters to the space of image parameters. The source is described by its magnitude M and radius a, while the image is represented by its apparent magnitude M_{ap} and apparent angle A_{ap}. Note that the magnitude scales are inverted in the two planes (the object plane being given with Arp's coordinates).



Detection limits in the image plane (right) mapped back into the object plane (left). There are detector brightness limits, PSF limits (optics), and detector (pixel) limits. The area U is unobservable. In the white area images are unidentifiable. Those in D are detectable.

Low surface brightness galaxies

- Low surface brightness galaxies lie in the regions of the object plane (for any given z) outside D, where they may be either undetectable (region U) or unidentifiable (white domain). As these limits depend on the cosmological model as well as the type of galaxy and the optics (PSF) and detector (pixel size, detection limits) theorists should take them into account.
- They require a minimum of 2 parameters (L,a) to be adequately modelled

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Feynmann, Gunn, **Kaiser**

Ricci focusing and Weyl focusing: Ehlers, Sachs, Penrose

B. Bertotti "The Luminosity of Distant Galaxies" ***Proc Royal Soc London.*** **A294, 195 (1966).**

$$d\theta/dv = -R_{ab}K^aK^b - 2\sigma^2 - \theta^2$$

$$d\,\sigma_{mn}/dv = -E_{mn}$$

Θ = expansion

σ = shear

R_{ab} = Ricci tensor, determined pointwise by matter

E_{ab} = Weyl tensor, determined non-locally by matter

Robertson-Walker observations:
zero Weyl tensor and non-zero Ricci tensor.

$$d\theta/dv = -R_{ab}K^aK^b - \theta^2$$

$$d\,\sigma_{mn}/dv = 0$$

Actual observations are best described by zero Ricci tensor and non-zero Weyl tensor

$$d\theta/dv = -\,2\sigma^2 - \theta^2$$

$$d\,\sigma_{mn}/dv = -\,E_{mn}$$

This averages out to FRW equations when averaged over whole sky Not obvious! It does not follow from energy conservation (Weinberg) - depends on how area distances average out. But supernova observations are preferentially where there is no matter

Observations and averaging

Dyer Roeder equations take matter into account but not shear: allows a fraction of the uniform density

C. C Dyer. & R C Roeder, “Observations in Locally Inhomogeneous Cosmological Models” Astrophysical Journal, Vol. 189: 167 (1974)

NB: must take shear and caustics into account

Note that how this works out depends on how dark matter is clustered. If it is uniform, Dyer-Roeder is good; if dark matter is clustered, it is not so good.

Why should it average out?

Weinberg: yes

Weinberg, S. (1976) "Apparent luminosities in a locally inhomogeneous universe." *The Astrophysical Journal* **208**: L1-L3.

Ellis Bassett Dunsby: no

Clarkson

Kibble and Lieu

Many others

➔ Kaiser paper with Peacock

Swiss-cheese (Einstein-Strauss) exact lumpy models can be used to test the observational effects

Exact vacuum static domains imbedded in an expanding universe model; no backreaction! (Birkhoff)

Example: *R. Kantowski "The Effects of Inhomogeneities on Evaluating the mass parameter Ω_m and the cosmological constant Λ" (1998) [astro-ph/9802208]*

"a determination of Ω_0 made by applying the homogeneous distance--redshift relation to SN 1997ap at $z = 0.83$ could be as much as 50% lower than its true value."

Weak Gravitational Lensing of Finite Beams

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In the weak-field regime and plane-parallel approximation, each point of the extended source is mapped to its image through the lens equation [33]

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \sum_{k=1}^{N} \varepsilon_k^2 \frac{\boldsymbol{\theta} - \boldsymbol{\theta}_k}{|\boldsymbol{\theta} - \boldsymbol{\theta}_k|^2}, \tag{1}$$

where $\boldsymbol{\beta}$, $\boldsymbol{\theta}$, and $\boldsymbol{\theta}_k$ denote, respectively, the angular positions, on the observer's celestial sphere, of the unlensed source, image, and lenses, as illustrated in Fig. 1. Besides, ε_k denotes the Einstein radius of the kth lens, with $\varepsilon_k^2 \equiv 4Gm_k(D - D_k)/(D_k D)$, where m_k is its mass and D_k its distance to the observer. The *weak-lensing regime* (stricter than the weak-field regime) applies if the physical size of the lenses is much bigger than their Einstein radii, so that: (i) there is only one image per point source, and (ii) $\varepsilon_k/|\boldsymbol{\theta} - \boldsymbol{\theta}_k| \ll 1$. The Einstein radii ε_k will thus be considered as small numbers in what follows.

Because they are two-dimensional vectors, $\boldsymbol{\beta}$, $\boldsymbol{\theta}$, and $\boldsymbol{\theta}_k$ can be represented by complex numbers s, z, and w_k, respectively, in terms of which the lens equation now reads

$$s = z - \sum_{k=1}^{N} \frac{\varepsilon_k^2}{z^* - w_k^*}, \tag{2}$$

where a star denotes complex conjugation. In the weak-lensing regime, this equation can be inverted order by order in ε_k^2 as $z(s) = s + \delta^{(2)}z(s) + O(\varepsilon^4)$, where $\delta^{(2)}z(s) \equiv \sum_{k=1}^{N} \frac{\varepsilon_k^2}{s^* - w_k^*}$. For the remainder of this Letter, we will work at order 2 in ε and drop the (2) superscript.

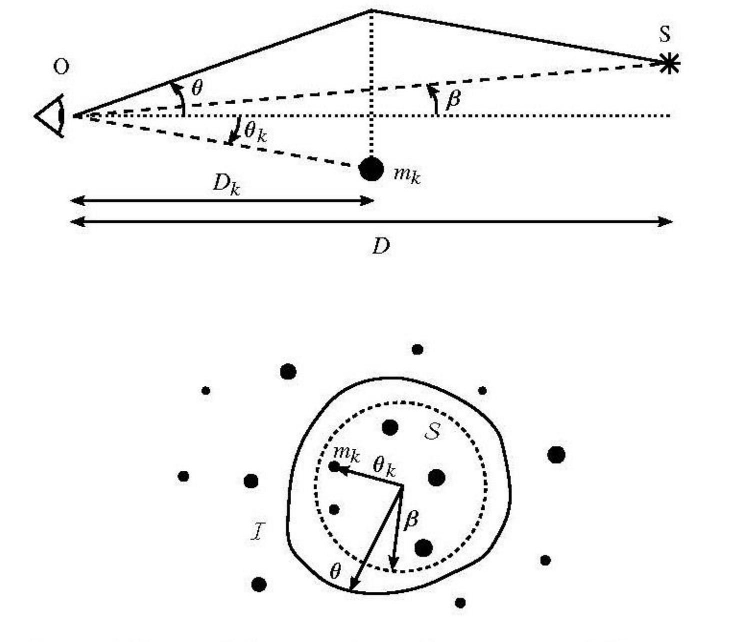

Figure 1. Top panel: Lensing of a single point source S. Bottom panel: Contours of the source S and image I, as seen by the observer.

From Weyl to Ricci.—We first investigate the lensing magnification of the extended source. We call S its contour in the complex plane, which is mapped to the image contour $\mathcal{I}$ via Eq. (2). The angular size of the image is, by definition,

$$\Omega = \frac{1}{2\mathrm{i}} \int_{\mathcal{I}} z^* \mathrm{d}z. \qquad (3)$$

$$\Omega = \frac{1}{2\mathrm{i}} \int_{S} s^* \mathrm{d}s + \frac{1}{2\mathrm{i}} \sum_{k=1}^{N} \varepsilon_k^2 \int_{\mathcal{I}} \frac{\mathrm{d}z}{z - w_k} - \frac{1}{2\mathrm{i}} \sum_{k=1}^{N} \varepsilon_k^2 \left[\int_{\mathcal{I}} \frac{z\mathrm{d}z}{(z - w_k)^2} \right]^* + O(\varepsilon^4). \qquad (4)$$

The first term on the right-hand side is the unlensed size of the source Ω_S, while the two other complex integrals can be computed using the residue theorem. Interestingly, only the lenses which are enclosed by the light beam, i.e. such that w_k lies in the region delimited by $\mathcal{I}$, contribute to the angular size at that order.

This result generalizes Ref. [29]. It shows that the area of a finite light beam, whose contour experiences Weyl curvature only, propagates like the area of an infinitesimal beam experiencing Ricci curvature only. Furthermore, this effective Ricci curvature is equal to the average Ricci curvature encountered inside the finite beam. In other words, the effect of a point mass m_k enclosed by the beam is identical to the effect of a homogeneous distribution of mass with surface density m_k/A_k, whatever its transverse position across the finite beam. This shows the ability of light beams to smooth out the matter distribution they enclose.

This property can be understood as a consequence of Gauss' theorem, which emphasizes the special status of the $1/r^2$ behavior of gravitation.

Conclusion.—In this Letter, we analyzed the properties of weak gravitational lensing beyond the infinitesimal-beam approximation. We addressed the Ricci-Weyl problem by showing that light beams are smoothing out the distribution of matter they enclose, at the scale of the beam's cross section. While only the lenses which are enclosed by the beam contribute to the convergence, at lowest order, any lens contribute to its distortions. Such distortions of the beam's morphology were decomposed over Fourier modes, elegantly expressed in terms of the position and mass of the lenses.

In particular, the standard weak-lensing shear was found to involve not only lenses out of the beam, but also interior lenses.

Shear caused by matter out of the beam will cause convergence inside the beam – contrary to what was stated. Issue: boundary of integral in (3)
Should have gone beyond image I

We see SN in preferred directions

Extremely thin pencil of light rays in vacuum

➔ Else we would not see the SN!

➔ Not a fair sample of the universe

(Mis)interpreting supernovae observations in a lumpy universe

Chris Clarkson, George F. R. Ellis, Andreas Faltenbacher, Roy Maartens, Obinna Umeh and Jean-Philippe Uzan

Mon. Not. R. Astron. Soc. **426,** 1121–1136 (2012)

Outcome depends on clustering of dark matter halos

And so on bias factor (is it constant?)

➔ We do not average over all directions. Preferred directions!!

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6 DISCUSSION AND CONCLUSIONS

The effect of the inhomogeneity of the matter distribution induces a dispersion of the magnification of SNIa and thus of the luminosity distance. We have argued that this effect has not been properly modelled for SNIa since the beam is very narrow and far below the scales resolved in any numerical simulation.

For the first time, we have attempted to quantify the probability distribution for narrow beams, using a combination of N-body simulations and a PS approach. For a narrow beam of fixed length, the PDF is non-Gaussian, peaked at densities below the cosmic mean, with a power-law tail, whose power depends on the diameter of the beam, describing the relatively few lines of sight which have an overdense mean. These PDFs contrast sharply with distributions based on using cubes of the same volume. These estimations are based on current N-body simulations which do not have the resolution to probe beams with a diameter $\lesssim 100\,h^{-1}$ kpc. Nevertheless, the trend is clear: narrow beams typically experience a lower than average density, and do not sample the cosmic mean density until their length approaches the Hubble scale. Based on our results, we estimate that significantly more than 75 per cent of the beams experience less than the mean density.

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Generically, then, distances to the same object will depend on the scale over which the light from the source smears the intervening matter distribution.

We have found that the old problem of modelling narrow beams remains unsolved. As different interpretations of the problem give conflicting yet significant effects, we believe this problem needs considerably more attention. This is important not only from a theoretical perspective, but to ensure precision cosmology delivers correct answers as well as precise ones.