Superfluid Dark Matter

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Introduction

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It faces some challenges at galactic scales.
MOND:
Milgrom ’83

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a = \begin{cases} 
  a_N & a_N \gg a_0 \\
  \sqrt{a_N a_0} & a_N \ll a_0
\end{cases}
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\[a_0 \sim H_0\]
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MOND fails on cosmological scales.
Unified Framework

Unification can be achieved through superfluidity.

\[ L_{MOND} = -2M_\text{pl}^3a_0 \left[ \left( \vec{\nabla} \phi \right)^2 \right]^{3/2} - \phi \rho \]

\[ L_{\text{superfluid}} = P(X) \]

with \( X \equiv \mu + \dot{\phi} - \left( \vec{\nabla} \phi \right)^2/2m \). The pressure \( P \) depends on the properties of the superfluid.
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\( P \) depends on the properties of the superfluid.
Bose-Einstein Condensate

Condensation takes place when

\[ \lambda_{dB} = \frac{1}{\sqrt{mT}} > \ell \]

At this point the particles become indistinguishable and the classical statistics breaks down.

The critical temperature for this phase transition is estimated to be

\[ T_c = \frac{n^2}{3m} \]
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At $T < T_c$

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BEC + Normal fluid
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BEC exhibits superfluidity if it is a condensate of interacting (repulsive) particles.
The sound waves of the superfluid (so called ’phonons’) propagate with the dispersion relation

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For the contact 2-point interactions with coupling constant \( \lambda \)

\[ c_s^2 = \frac{\lambda n}{m^3} \]
Superfluidity

For small momenta

\[ \omega_k = c_s k \]
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Landau’s criterion for superfluidity

\[ v < c_s \]
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This is the solution as long as

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\[ \Psi = v e^{i \tilde{\mu} t} \]

This is the solution as long as

\[ \tilde{\mu}^2 - m^2 - \lambda v^2 = 0 \]

\( v \) is fixed by the concentration

\[ n = 2 \tilde{\mu} v^2 \]
Fluctuations

\[ \Psi = (v + \rho)e^{i(\tilde{\mu}t + \varphi)} \]
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Easy to show that

$$m_\rho = 2m \quad \text{and} \quad m_\varphi = 0$$
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We can integrate out \( \rho \) to get

\[ \mathcal{L} \propto X^2 \quad \text{with} \quad X \equiv \mu + \dot{\phi} - \frac{\vec{\nabla} \phi^2}{2m} \]
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For generic potential we get a generic function of $X$. 
Unified Framework

\[ \mathcal{L}_{\text{MOND}} \sim \left[ (\vec{\nabla} \varphi)^2 \right]^{3/2} \]

vs

\[ \mathcal{L}_{\text{superfluid}} = P(X) \quad \text{with} \quad X \equiv \mu + \dot{\varphi} - \frac{(\vec{\nabla} \varphi)^2}{2m} \]

\( P \) depends on the properties of the superfluid.
Fractional Powers?

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Unitary Fermi Gas is described by

\[ \mathcal{L}_{\text{UFG}} \sim X^{5/2} \]
\( X^{3/2} ? \)

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\[ X^{3/2}? \]

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EFT for phonons is

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BEC of Dark Matter

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Thermalization requirement

\[ \Gamma \gtrsim t_{\text{dyn}}^{-1} \]
Density Profile

At virialization we expect NFW profile

As density increases, it becomes easier to satisfy the condensation criteria.
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As density increases, it becomes easier to satisfy the condensation criteria.

Therefore, the region above some critical density will transform into the superfluid.
Density Profile

Galaxies: $R_{\text{core}} > R / 3 \Rightarrow m < 1.5 \left( \frac{\sigma}{m_0 \cdot 0.5 \text{ cm}^2/\text{g}} \right)^{1/4} \text{eV}$

Clusters: $R_{\text{core}} < R / 10 \Rightarrow m > 0.75 \left( \frac{\sigma}{m_0 \cdot 0.5 \text{ cm}^2/\text{g}} \right)^{1/4} \text{eV}$
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Cosmology:

DM with eV mass must be produced out of equilibrium. For instance, through axion-like vacuum displacement mechanism.
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$$\left( \frac{T}{T_c} \right)_{\text{cosmo}} \sim 10^{-28} \quad \text{vs} \quad \left( \frac{T}{T_c} \right)_{\text{MW}} \sim 10^{-2}$$
Superfluid Properties:

Low energy degrees of freedom are phonons.
In general

\[ \mathcal{L} = P(X), \quad \text{with} \quad X \equiv \mu + \dot{\phi} - \frac{(\nabla \phi)^2}{2m} \]
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We conjecture that the superfluid has MOND-like action

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To mediate MOND force, phonons must couple to baryons

\[ \mathcal{L}_{\text{int}} = -\alpha \frac{\Lambda}{M_{\text{pl}}} \varphi \rho_b \]
Superfluid Properties:

The pressure of the condensate is

\[ P(\mu) = \Lambda (m\mu)^{3/2} \]
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The equation of state is

\[ P = \frac{\rho^3}{12\Lambda^2 m^6} \]

The sound speed is given by \( c_s = \sqrt{\frac{2\mu}{m}} \)
Core Profile:

Assuming hydrostatic equilibrium

\[ \rho(r) \simeq \rho_0 \cos^{1/2} \left( \frac{\pi}{2} \frac{r}{R} \right); \quad r \leq R \]
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For fiducial values \( m = \text{eV} \) and \( \Lambda = 0.04 \text{ meV} \), for \( M_{\text{DM}} = 10^{12} M_\odot \), we have

\[ R \approx 120 \text{ kpc} \]

The superfluid scenario provides the simple resolution to the cusp-core and "too-big-to-fail" problems.
In order to have \( w \ll 1 \) at matter radiation equality, we need \( \Lambda \gg 0.1 \) eV. This is five orders of magnitude larger than the fiducial value \( \Lambda = 0.04 \) meV assumed in galaxies. Recall that \( (T/T_c)_{\text{cosmo}} \approx 10^{-28} \) \( (T/T_c)_{\text{MW}} \approx 10^{-2} \).
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Recall that

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Phonon-Mediated Force Between Baryons

In the limit of large phonon gradients

\[ a_\phi(r) \simeq \sqrt{\frac{\alpha^3 \Lambda^2}{M_{Pl}}} \frac{G_N M_b(r)}{r^2} \]
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Closer Look at Rotation Curves

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\[
a_{\text{phonons}} < a_{N-\text{total}} < a_{\text{MOND}}
\]

\[
a_0^{\text{superfluid}} < a_0^{\text{MOND}}
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\[ a_0^{\text{superfluid}} = 0.73a_0^{\text{MOND}} \]
Rotation Curves

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\[ M_b = 3.4 \times 10^9 M_\odot \]

\[ M_{\text{total}} = 10 M_b \]

\[ R_b = 12 \text{kpc} \]

\[ R_{\text{core}} = 49 \text{kpc} \]

\[ R_{\text{vir}} = 49 \text{kpc} \]

\[ M_b = 1.6 \times 10^{11} M_\odot \]

\[ M_{\text{total}} = 10 M_b \]

\[ R_b = 60 \text{kpc} \]

\[ R_{\text{core}} = 65 \text{kpc} \]

\[ R_{\text{vir}} = 126 \text{kpc} \]
Validity of EFT and the Solar System

MOND regime corresponds to large phonon gradients

\[ \frac{\varphi'^2}{2m} \gg \mu \]

In terms of superfluid velocity \( v_s = |\vec{\nabla}\varphi|/m \), this becomes

\[ v_s \gg c_s \]

Thus, violating Landau’s criterion.

This is not surprising, since we are interested in generating large coherent phonon background.

High derivative terms need to be small. This is so as long as \( r \gg \Lambda_s^{-1} \).
Validity of EFT and the Solar System

We should also verify that

\[ v_s \ll v_c \equiv \left( \frac{\rho}{m^4} \right)^{1/3} \]

Leading to

\[ r \gg 0.2 \left( \frac{M_b}{10^{11} M_\odot} \right)^{1/2} \left( \frac{m}{\text{eV}} \right)^{-1/2} \left( \frac{\Lambda}{\text{meV}} \right)^{-5/6} \text{ kpc} \]

Applied to the Sun

\[ r \gg 250 \left( \frac{m}{\text{eV}} \right)^{-1/3} \left( \frac{\Lambda}{\text{meV}} \right)^{-5/9} \text{ AU} \]
Gravitational Lensing

We might need to couple baryons to

\[ \tilde{g}_{\mu\nu} \simeq g_{\mu\nu} - \frac{2\alpha}{M_{\text{pl}}} \varphi \left( \gamma g_{\mu\nu} + (1 + \gamma) u_\mu u_\nu \right) \]

In TeVeS \( \gamma = 1 \).
Gravitational Lensing

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In TeVeS \( \gamma = 1 \).

In our case we have DM, unlike TeVeS, and hence we may even get away with \( \gamma = -1 \).
In general, the outcome depends on relative fraction of superfluid vs normal components in the clusters.
Merging Clusters:

In general, the outcome depends on relative fraction of superfluid vs normal components in the clusters.

If the infall velocity is sub-sonic, the superfluid components should pass through each other with negligible friction, however the normal components should be slowed down.
Summary

Assumptions:

- Bose-Einstein condensation
- Superfluid with equation of state $p \sim \rho^3$
- Phonon-mediated force
- Temperature dependance of parameters

Results: DM halos are superfluids at finite temperature, with interesting phenomenology.

Next Step: Finding a viable microscopic theory.
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