Baryogenesis from L-violating Higgs doublet decay

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       arXiv:1705.00016
Baryogenesis via leptogenesis

- very highly motivated: same origin as neutrino masses
- very natural at high scale: a series of numerical coincidences which make it particularly efficient but very difficult to test
- clearly possible at low scale: if seesaw states have a quasi-degenerate mass spectrum and/or if large cancellation among Yukawa couplings
- this talk: new way at low scale: total lepton number violating Higgs doublet decay into \(\sim 0.1\text{-}100\) GeV right-handed neutrinos
Leptogenesis relevant scales for low $m_N$

usual leptogenesis: $m_N >> T_{Sphaler.} > m_{H,L}$: leptogenesis from $N \rightarrow LH$ decay

creation of L asymmetry at $T \sim m_N >> T_{Sphaler.} \Rightarrow$ B asymmetry

resonant propagator if $m_{N_j} \sim m_{N_i} \Rightarrow \sim$TeV scale leptogenesis

very low scale leptogenesis: $T_{Sphaler.} > m_H >> m_{N,L}$

creation of L asymmetry at $T > T_{Sphaler.} >> m_N \Rightarrow \neq$ regime

thermal effects are fully relevant: $T > T_{Sphaler.} > m_H >> m_{N,L}$

$$m_H^2(T) = m_H^2 + c_H \cdot T^2 \quad m_L^2(T) = m_L^2 + c_L \cdot T^2 \quad m_N^2(T) = m_N^2 + c_N \cdot T^2$$

$N \rightarrow LH$ forbidden but $H \rightarrow NL$ allowed
Temperatures allowing the $N \rightarrow LH$ and $H \rightarrow NL$ decays

$T_{Sphaler.}$

$m_N > m_H + m_L$

$m_H > m_N + m_L$

$H \rightarrow NL$ leptogenesis from this region?
L asymmetry production from $H \rightarrow NL$ decay

2 issues at first sight:

1) out-of-equilibrium decay? $\quad \leftarrow$ 3rd Sakharov condition

$H$ decaying particle is in deep thermal equilibrium at $T > T_{Sphaler}$. but $N$ in decay product is not necessarily in thermal equilibr.

\[
\frac{dn_N}{dt} \propto (n^{eq}_N - n_N) \cdot \Gamma_{H \rightarrow NL}
\]

\[
H \rightarrow NL \quad NL \rightarrow H
\]
L asymmetry production from $H \rightarrow NL$ decay

2) Absorptive part for CP violation?

$m_H + m_L > m_N \Rightarrow$ no absorptive part?

but only for $T = 0$!

finite $T$ corrections: thermal cut: if $H$ or $L$ comes from the thermal bath the cut is kinematically allowed

$\Gamma_N(T')$ (calculated in Kadanoff Baym formalism)

$\Pi^\alpha\beta(q_N) = \Pi^\alpha_>(q_N) + \Pi^\alpha_< (q_N)$

$\exists -2 \int \frac{d^4p_l}{(2\pi)^4} \frac{d^4p_H}{(2\pi)^4} (2\pi)^4 \delta^4(q_N - p_l - p_H) \langle Y_N^\dagger Y_N \rangle_{\alpha\beta} [P_L S^l_>(t, p_l) P_R \Delta^H_>(t, p_H)]$

$\Gamma_N(T) = \frac{1}{8\pi} m_{N_2} \langle Y_N Y_N^\dagger \rangle_{22} \cdot \frac{p \cdot L_N}{q_N \cdot p_l}$

$L_N = 16\pi \int \frac{d^3p_l}{(2\pi)^3} \frac{d^3p_H}{(2\pi)^3} (2\pi)^2 \delta^4(q_N - p_l - p_H) (f^{FD}_l + f^{BE}_l) p_l$
Total L number violating CP asymmetry

\[ \varepsilon_{CP} = \frac{\text{Im}[(Y_N Y_N^\dagger)_{22}]}{(Y_N Y_N^\dagger)_{11}(Y_N Y_N^\dagger)_{22}} \cdot \frac{2 \Delta m_N^0 \Gamma_N(T)}{4 \Delta m_N(T)^2 + \Gamma_N(T)^2} \]

with thermal mass splitting: \( \Delta m_N(T) \simeq \Delta m_N^0 + \frac{\pi T^2}{4 m_N^2} \Gamma_{22} \sqrt{1 + \frac{\Gamma_{11}^2}{\Gamma_{22}^2}} + 4 \frac{\Gamma_{12}^2}{\Gamma_{22}^2} \)

\[ \Gamma_{ij} \equiv m_N(Y_N Y_N^\dagger)_{ij}/(8\pi) \]

Boltzmann equations:

\[ \frac{n_H H_N}{z} \frac{d\eta_N}{dz} = \left( 1 - \frac{\eta_N}{\eta_N^{\text{eq}}} \right) \left[ \gamma_D + 2(\gamma_{Hs} + \gamma_{As}) + 4(\gamma_{Ht} + \gamma_{At}) \right] + \gamma_D \left[ \left( \frac{\eta_N}{\eta_N^{\text{eq}}} - 1 \right) \varepsilon_{CP}(z) - \frac{2}{3} \eta_L \right] - \frac{4}{3} \eta_L \left[ 2(\gamma_{Ht} + \gamma_{At}) + \frac{\eta_N}{\eta_N^{\text{eq}}} (\gamma_{Hs} + \gamma_{As}) \right] \]

\[ \eta_N \equiv n_N/n_\gamma \]

\[ z \equiv m_N/T \]
Results for the case where the N have thermalized

if N thermalized by large $Y_N$ Yukawas or other interaction (e.g. a $W_R$) before an asymmetry is produced

the lower is $m_N$, the later it goes out-of-equilibrium, the more it will be in equilibrium at $T > T_{Sphaler}$.  

lower bound on $m_N$  

$m_N > 2.2 \text{ GeV}$  

if only $N \rightarrow LH$ decay we get: $m_N > 50 \text{ GeV}$

requires that at least 2 of the $N$ have quasi-degenerate masses
Results for the case where the N have not thermalized

- if no extra interaction thermalizing $N$, no thermalization is much more natural than in ordinary leptogenesis: thermalization at $T > T_{Sphaler}$. $\gg m_N$
- requires much larger $Y_N$ Yukawas than in ordinary leptogenesis at $T \sim m_N$

\[ \tilde{m} \equiv \frac{Y_N Y_N^\dagger v^2}{2m_N} \]

\[ \tilde{m} \gg 10^{-3} \text{ eV} \]

\[ \tilde{m} \gtrsim 10^{-3} \text{ eV} \]

- for $H \to NL$ decay, to start from no $N$ in the thermal bath boosts the asymmetry production, unlike for ordinary $N \to LH$ leptogenesis

\[ H \to NL: \text{many } H \text{ to decay and produce the asymmetry but few } N \text{ to } NL \to H \text{ inverse decay} \]

\[ n_{N}^{eq} - n_N \sim n_{N}^{eq} \gg n_N \]
**Results for the case where the N have not thermalized**

- For example, for $m_N \sim 10 \text{ GeV}$ and $\tilde{m} \sim 0.1 \text{ eV}$, one needs $\Delta m_N^0 / m_N \lesssim 10^{-5}$.
- Leptogenesis for $m_N$ as low as $\sim 20 \text{ MeV}$ is possible (but BBN concerns).
- In all cases: asymmetry production at $T$ just above $T_{Sphaler}$.

\[ \text{no dependence on UV physics!} \]
that the minimum level of mass-degeneracy required is of magnitude smaller than in ordinary TeV-scale resonant nesses, which requires spectively. Fig. 5b and 5c show that successful leptoge-

T in (10), with zero number density of RH neutrinos at not depend on the reheating temperature as long as this approximately constant: and small shortly before sphaleron decoupling because for

CP-asymmetry far from max-

m

RH neutrinos at

due to the sphaleron cut. Note that this is di-

malized before the lepton asymmetry is produced. If in-

occurs, even if

large washout e

are in the thermal bath, the more

mass quasi-degeneracy about two orders

bound because in the low-

m

is true only if one assumes that the N species has ther-

thermal equilibrium at

III. THE NON-THERMALIZED CASE: AN EFFICIENT LOW-SCALE MECHANISM

As explained above, the bound of Eq. (14) holds for 

H decays slightly before the sphalerons decouple does

by a factor of about 2.

The fact that it is in

unlike

in

after reheating renders leptogenesis more di-

in non-thermalized with the ARS oscillation one [19] (which also relies on

ARS one, for

by the density-matrix formalism used to study the ARS practice also that, although the L-violating e

no other interactions below the reheat-

region that FCC should probe if constructed

region that SHiP should probe

Testability!
Two important comparisons to do

- for \( m_N \sim \text{GeV} \): well-known baryogenesis mechanism in seesaw model: baryogenesis from right-handed neutrino oscillations: \`ARS'\ mechanism
  
  \[
  \begin{align*}
  \text{Akhmedov, Rubakov, Smirnov 98'} \\
  \text{Asaka, Shaposhnikov 05'; Shaposhnikov 08'} \\
  \text{Drewes, Garbrecht 11'} \\
  \text{Canetti, Drewes, Frossard, Shaposhnikov 13'} \\
  \text{Hernandez, Kekic, Lopez-Pavon, Racker, Rius 15'} \\
  \ldots
  \end{align*}
  \]

  comparison of ARS with L-violating Higgs decay setup???

- to compute evolution of asymmetries with thermal effects: another well-known formalism: density matrix formalism

  comparison of results of decay formalism and density matrix formalism???
ARS vs \( L \)-violating \( H \)-decay scenarios

ARS is based on processes where there is no \( m_N \) mass insertions:

- only \( N \to L \) and \( \bar{N} \to \bar{L} \) transitions
- \( \text{assigning } L = 1 \) to \( N \) and \( L = -1 \) to \( \bar{N} \), all processes conserve \( L \):

\[
\sum_{i=e,\mu,\tau} (n_{L_i} - n_{\bar{L}_i}) + \sum_{\alpha} (n_{N_{\alpha}} - n_{\bar{N}_{\alpha}}) = 0
\]

\( \Rightarrow \) at \( \mathcal{O}(h^4) \): SM lepton number and \( N \) lepton number are separately conserved:

\[
\sum_{i=e,\mu,\tau} (n_{L_i} - n_{\bar{L}_i}) = 0 \quad \sum_{\alpha} (n_{N_{\alpha}} - n_{\bar{N}_{\alpha}}) = 0
\]

but flavour lepton number is not conserved: \( \delta n_i \equiv n_{L_i} - n_{\bar{L}_i} \neq 0 \)

\( \Rightarrow \) at \( \mathcal{O}(h^6) \): if for example Yukawa for electron much smaller than for muon:

- \( n_{L\mu} - n_{\bar{L}\mu} \) strongly washed-out
- \( n_{L\epsilon} - n_{\bar{L}\epsilon} \) much less washed-out

\( \Rightarrow \)

\[
\sum_{i=e,\mu,\tau} (n_{L_i} - n_{\bar{L}_i}) = -\sum_{\alpha} (n_{N_{\alpha}} - n_{\bar{N}_{\alpha}}) \neq 0
\]

converted to \( B \) asym by sphalerons

not converted to \( B \) asym by sphalerons

\( \Rightarrow \) baryon asymmetry!
ARS vs $L$-violating $H$-decay scenarios

2 ≠ contributions:

ARS gives a $L$ conserving contribution (no $m_N$ insertion) arising at $\mathcal{O}(h^6)$ which vanishes if only one lepton flavour

Higgs decay above gives a $L$ violating contribution (based on $m_N$ insertion) arising at $\mathcal{O}(h^4)$ already with a single lepton flavour

where to find the LV contribution in density matrix formalism?
Density matrix formalism

$N_{R\alpha}$ quantum system is described by density matrix: $n^{N}_{\alpha\beta} \equiv \langle a^{+}\dagger a^{+} \rangle = Tr(\rho a^{+}\dagger a^{+})$

$\bar{N}_{R\alpha}$ quantum system is described by density matrix: $n^{\bar{N}}_{\alpha\beta} \equiv \langle a^{-}\dagger a^{-} \rangle = Tr(\rho a^{-}\dagger a^{-})$

$n^{N}_{\alpha\alpha} = n^{N}_{\alpha} = $ number density of $N_{\alpha}$ states

$n^{N}_{\alpha\beta} = $ coherence between $N_{\alpha}$ and $N_{\beta}$ states

$\Rightarrow$ evolution of density matrix:

$$\frac{d}{dt} n^{N}_{\alpha\beta}(k, t) = i \langle [H_{0}^{N}, n^{N}_{\alpha\beta}(k, t)] \rangle - \frac{1}{2} \int_{-\infty}^{\infty} dt' \langle [H_{\text{int}}(t'), [H_{\text{int}}(t), n^{N}_{\alpha\beta}(k, t)]] \rangle_t$$

oscillation term

interaction term

$H_{\text{int}} = h_{l\alpha} \bar{L}_{i} \bar{H} P_{R} N_{\alpha} + h.c.$

$\Rightarrow$ $H_{\text{int}} \cdot H_{\text{int}}$ terms in $a^{+}\dagger a^{+} \rightarrow n^{N}_{\alpha\beta}$

$\Rightarrow$ $H_{\text{int}} \cdot H_{\text{int}}$ terms in $a^{-} a^{-}\dagger \rightarrow 1 - n^{N}_{\alpha\beta}$

$$\frac{d}{dt} n^{N}_{\alpha\beta}(k) = -i \left[ E_{N}, n^{N}(k) \right]_{\alpha\beta} - \frac{1}{2E_{N}} \left( \frac{1}{2} \{ \Gamma^{>}(k), n^{N}(k) \} - \frac{1}{2} \{ \Gamma^{<}(k), 1 - n^{N}(k) \} \right)_{\alpha\beta},$$

$N + \bar{L} \rightarrow H \quad H \rightarrow N + \bar{L}$
ARS contribution in density matrix formalism

keeping only the transitions where there is no $m_N$ mass insertions because the asymmetry is produced at $T >> T_{sphaler.} >> m_N$

if mass insertion: $m_N^2/T^2$ suppression

$$\Gamma_{\alpha\beta}^{\lesssim}(k) = -i \text{tr} \left\{ P_R u+(k) \bar{u}+(k) P_L \Sigma^{\lesssim}_{\alpha\beta}(k) \right\}$$

with:

$$-i \Sigma^{\lesssim}_{\alpha\beta}(k) = \int \frac{d^4 p}{(2\pi)^4} \int \frac{d^4 q}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(q - k - p) \ i S_i^{\lesssim}(-p) i \Delta^{\lesssim}(-q) h_{l\alpha}^* h_{l\beta}$$

Wightman propagator of $H$

Wightman propagator of $L$

$$\frac{d n_{\alpha\beta}^N}{dt} = -i [\mathcal{E}_N, n_N^N(k)]_{\alpha\beta} - \frac{1}{2} \left\{ \gamma^{LC}, \frac{n_N^N}{n_{eq}^N} - I \right\}_{\alpha\beta} + \frac{\delta n_{l}^{L}}{2n_{eq}^{L}} \left( (\gamma^{LC}_{WQ,l}) + \frac{1}{2} \left\{ \gamma^{LC}_{WC,l}, \frac{n_N^N}{n_{eq}^N} \right\} \right)_{\alpha\beta}$$

LC production rate $\propto T^4$

$$\frac{d \delta n_{l}^{L}}{dt} = \frac{1}{n_{eq}^{L}} \text{tr} \left\{ (\gamma^{LC}_{l}) n_N^N \right\} - \frac{1}{n_{eq}^{N}} \text{tr} \left\{ (\gamma^{LC*}_{l}) \bar{n}_N^N \right\}$$

LC washout rates $\propto T^4$

$$-\frac{\delta n_{l}^{L}}{n_{eq}^{L}} \text{tr} \left\{ \gamma^{LC}_{WQ,l} \right\} - \frac{\delta n_{l}^{L}}{2n_{eq}^{L}} \frac{1}{n_{eq}^{N}} \text{tr} \left\{ n_N^N (\gamma^{LC}_{WC,l} ) \right\} - \frac{\delta n_{l}^{L}}{2n_{eq}^{L}} \frac{1}{n_{eq}^{N}} \text{tr} \left\{ \bar{n}_N^N (\gamma^{LC*}_{WC,l} ) \right\}$$
keeping also the terms with mass insertion, $\propto m_N^2$

$N$ to $\bar{L}$ transition instead of $N$ to $L$ transition

$$\Gamma_{\alpha\beta}(k) \equiv +i \text{tr} \left\{ P_R v_+ (k) \bar{v}_+ (k) P_L \Sigma_{\beta\alpha} (-k) \right\}$$

$$\frac{d n_{\alpha\beta}^N}{dt} = -i [\mathcal{E}_N, n_N^N (k)]_{\alpha\beta} - \frac{1}{2} \left\{ \gamma^{LC} + \gamma^{LV}, \frac{n_N^N}{n_{eq}} - I \right\}_{\alpha\beta}$$

$$\frac{d \bar{n}_{\alpha\beta}^N}{dt} = -i [\mathcal{E}_N, \bar{n}_N^N (k)]_{\alpha\beta} - \frac{1}{2} \left\{ \gamma^{LC*} + \gamma^{LV*}, \frac{\bar{n}_N^N}{n_{eq}} - I \right\}_{\alpha\beta}$$

$$\frac{d \delta n_l^L}{dt} = \frac{1}{n_{eq}} \text{tr} \left\{ \left( \gamma^{LC} - \gamma^{LV} \right) n_N^N \right\}$$

$$\frac{d \delta n_l^L}{dt} = \frac{1}{n_{eq}} \text{tr} \left\{ \left( \gamma^{LC*} - \gamma^{LV*} \right) \bar{n}_N^N \right\}$$

$$\frac{d \delta n_l^L}{dt} = \frac{1}{n_{eq}} \text{tr} \left\{ \left( \gamma^{LC}_l - \gamma^{LV}_l \right) n_N^N \right\}$$

$$\frac{d \delta n_l^L}{dt} = \frac{1}{n_{eq}} \text{tr} \left\{ \left( \gamma^{LC*}_l - \gamma^{LV*}_l \right) \bar{n}_N^N \right\}$$

$$\gamma^{LV} = \frac{g_{iso}}{4\pi^2} \int_0^\infty dk \frac{k}{e^{k/T} + 1} \frac{m_N^2}{32\pi k^3} \times \int_{E^*}^{\infty} dE (4E_k + M_L^2 - M_H^2) \left( \frac{1}{e^{E/k} + 1} \right) \frac{1}{e^{E/k} + 1} - 1 \right\} \approx 3.35 \times 10^{-3} m_N^2 T^2 h_{l_{\alpha}} h_{l_{\beta}}$$

$$\gamma^{LV} = \frac{g_{iso}}{4\pi^2} \int_0^\infty dk \frac{m_N^2}{32\pi k^3} \times \int_{E^*}^{\infty} dE (4E_k + M_L^2 - M_H^2) \left( \frac{1}{e^{E/k} + 1} \right) \frac{1}{e^{E/k} + 1} - 1 \right\} \approx 5.49 \times 10^{-4} m_N^2 T^2 h_{l_{\alpha}} h_{l_{\beta}}$$

T.H., Teresi 17'

similar to Shaposhnikov '08 and Ghiglieri, Laine 17'
Analytical solution for the LC and LV contribution for weak washout

\[ Y_{LC} \simeq -18.5 \times (\alpha^{LC})^2 \alpha_W^{LC} \frac{M_0^{7/3}}{T_c(\Delta m_N^2)^{2/3}} \times (h^\dagger h)_{11}(h^\dagger h)_{22} \sum_l \delta_l^{LC} (h h^\dagger)_{ll} \]

CP-violating Yukawa combination which leaves the SM total lepton number unchanged

\[ \delta_l^{LC} = \frac{\text{Im}[h_{i1}^* h_{i2} (h^\dagger h)_{12}]}{(h^\dagger h)_{11}(h^\dagger h)_{22}} \]

\[ Y_{LV} \simeq 7.9 \times \alpha^{LC} \alpha^{LV} \frac{M_0}{T_c} \frac{m_N^2}{\Delta m_N^2} (h^\dagger h)_{11}(h^\dagger h)_{22} \delta^{LV} \]

CP-violating Yukawa combination which break total lepton number

\[ \delta_l^{LV} = \frac{\text{Im}[h_{i1}^* h_{i2} (h^\dagger h)_{12}]}{(h^\dagger h)_{11}(h^\dagger h)_{22}} \]

⇒ LV vs LC contributions:

- \( \mathcal{O}(h^4) \) instead of \( \mathcal{O}(h^6) \) for the LC contribution
- suppressed by 2 rates instead of 3 rates for the LC contribution
- but \( m_N^2 \) suppression with different \( \Delta m_N^2 \) and \( M_0 \) dependence

⇒ all in the various factors compensate each other more or less with dominance of one or the other contribution depending on the parameters.
Numerical results: comparison of decay and density matrix formalisms for the LV contribution

- with only one lepton flavour: no ARS, only LV contribution

\[ Y_B = \frac{n_B}{s} \] contour plot \quad \Delta m_N/m_N = 10^{-10} \quad Y_B = \frac{n_B}{s} \] contour plot \quad \Delta m_N/m_N = 10^{-8.5}

⇒ qualitative or even quantitative agreement:

- except for small \( m_N \): different thermal masses taken
- except for large \( \tilde{m} \): washout suppression too big in decay formalism because doesn’t take into account formation of \( N - \bar{N} \) asymmetries
- in decay formalism the \( H \) is decaying “at rest” unlike in density matrix formalism
Numerical results: comparison of LC and LV contributions in matrix density formalism

\[ Y_B = \frac{n_B}{s} \] contour plot: full LC+LV result

\[ \Delta m_N/m_N = 10^{-10} \]

ratio of LV+LC over LC

dominance of LV= - for ``seesaw'' expected Yukawa couplings
- for very large Yukawas: less washout for LV than for LC !

\[ \propto m_N^2 T^2 \ll \propto T^4 \]

- the smaller \( \Delta m_N/m_N \) the more LV dominates
- the larger \( m_N \) the more LV dominates
Numerical results: comparison of LC and LV contributions in matrix density formalism

$$Y_B = \frac{n_B}{s}$$ contour plot: full LC+LV result

ratio of LV+LC over LC

$\Delta m_N/m_N = 10^{-8}$

dominance of LV= - for "seesaw" expected Yukawa couplings
- for very large Yukawas: less washout for LV than for LC!

$$\propto \frac{m_N^2}{T^2} \ll \propto T^4$$
- the smaller $\Delta m_N/m_N$ the more LV dominates
- the larger $m_N$ the more LV dominates
Dominance of the LV contribution for low reheating temperatures

LV contribution produced at lower temperature than the ARS-LC contribution due to the $m_N^2/T^2$ relative factor
Need to incorporate other processes for a full quantitative asymmetry calculation

top quark scattering processes, gauge scattering processes, ... have all a $H \rightarrow LN$ transition as building block

same mechanism expected to be operative

Besak, Bodeker 12'
see also Ghiglieri, Laine 17'
Summary

In usual leptogenesis decay formalism the L violating $H \rightarrow NL$ decay can easily lead to enough baryon asymmetry for $m_N < m_H$

- in type-I seesaw model with nothing else
- thanks to thermal effect leading to $N$ self-energy thermal cut
- from total L number violating CP asymmetries: no need for flavour interplay
- at electroweak scale temperatures: $T \gtrsim T_{\text{Sphaler.}}$
- with boosted production if no $N$ to begin with
- in a testable way (SHIP,...) for part of the parameter space

We have confirmed these results in density matrix formalism...

- both ARS-LC and LV contributions can dominate baryogenesis depending on parameters
baryon asymmetries obtained for 3 values of $\Delta m^0_N/m_N$
\( \gamma_{\alpha\beta}^{LC} = \int d\Pi_{PS} n_{eq}^N(k)(n_{eq}^L(p) + n_{eq}^H(q)) \times \text{tr} \{ P_R u_+(k) \bar{u}_+(k) P_L \bar{p} \} \ h_{l\alpha}^* h_{l\beta} \)

\( = \frac{g_{iso}}{4\pi^2} \int_0^\infty dk \frac{k}{e^{k/T} + 1} \frac{M_H^2 - M_L^2}{8\pi k} \times \int_{E^*}^\infty dE \left( \frac{1}{e^{E/T} + 1} + \frac{1}{e^{E+k/T} - 1} \right) h_{l\alpha}^* h_{l\beta} \)

\( \simeq 3.26 \times 10^{-4} T^4 h_{l\alpha}^* h_{l\beta} \)

\( \gamma_{\alpha\beta}^{WQ} = \frac{g_{iso}}{4\pi^2} \int_0^\infty dk k \frac{M_H^2 - M_L^2}{8\pi k} \times \int_{E^*}^\infty dE \frac{1}{e^{E/T} + 1} \frac{1}{e^{E+k/T} - 1} h_{l\alpha}^* h_{l\beta} \simeq 1.05 \times 10^{-4} T^4 h_{l\alpha}^* h_{l\beta} \)

\( \gamma_{\alpha\beta}^{WC} = \frac{g_{iso}}{4\pi^2} \int_0^\infty dk \frac{k}{e^{k/T} + 1} \frac{M_H^2 - M_L^2}{8\pi k} \int_{E^*}^\infty dE \frac{1}{e^{E/T} + 1} h_{l\alpha}^* h_{l\beta} \simeq 1.86 \times 10^{-4} T^4 h_{l\alpha}^* h_{l\beta} \)

and similarly for \( \bar{N} \).
Density matrix formalism: final evolution equation for $\delta n_l$

with no Majorana mass insertion

$\Rightarrow$ at $\mathcal{O}(h^4)$:

$$\frac{d \delta n_l^L}{dt} = \frac{1}{n_{eq}^N} \text{tr}\{(\gamma_l^{LC}) n^N\} - \frac{1}{n_{eq}^N} \text{tr}\{ (\gamma_l^{LC*}) \bar{n}^N\}$$

for not too large Yukawa coupling these equations can be analytically solved order by order in the Yukawa couplings

$\Rightarrow$ at $\mathcal{O}(h^4)$:

$$Y_i(z) \simeq 4 (\kappa_{LC}^l)^2 \rho_{eq} (h^\dagger h)_{11} (h^\dagger h)_{22} \delta_l^{LC} \int_0^z \text{Im} f(z')dz' \quad f(z) \equiv \int_0^z dz' e^{i \frac{\Delta m_{31}^2}{2 \mu_{osc}} (z'^2 - z^2)}$$

$$\delta_l^{LC} = \frac{\text{Im}[h_{11}^* h_{22} (h^\dagger h)_{21}]}{(h^\dagger h)_{11} (h^\dagger h)_{22}}$$

CP-violating Yukawa combination which leaves the SM total lepton number unchanged:

$$\delta n_{l_i} \equiv n_{L_i} - n_{\bar{L}_i} \neq 0 \quad \text{but} \quad \sum_{i=e,\mu,\tau} \delta n_{l_i} = 0 \Rightarrow \sum_{i=e,\mu,\tau} Y_i = 0$$

$\Rightarrow$ no contribution at $\mathcal{O}(h^4)$!
Density matrix formalism: final result with no Majorana mass insertion

\[ Y_{LC} \simeq -18.5 \times (\alpha^{LC})^2 \alpha_W^{LC} \frac{M_0^{7/3}}{T_c(\Delta m^2_N)^{2/3}} \times (h^\dagger h)_{11} (h^\dagger h)_{22} \sum_l \delta_l^{LC} (hh^\dagger)_{ll} \]

what happen’s is that since there is no \( m_N \) mass insertions in the processes nowhere, all processes conserve total lepton number: assigning \( L = 1 \) to \( N_R \) and \( L = -1 \) to \( \overline{N_R} \), all processes conserve \( L \)

\[ \Rightarrow \text{ at } O(h^4) \text{: SM lepton number and } N \text{ lepton number are separately conserved:} \]

\[ \sum_{i=e,\mu,\tau} (n_{L_i} - n_{\bar{L_i}}) = 0 \quad \sum_{\alpha} (n_{N_\alpha} - n_{\bar{N}_\alpha}) = 0 \]

but flavour lepton number is not conserved: \( \delta n_{l_i} \equiv n_{L_i} - n_{\bar{L_i}} \neq 0 \)

\[ \Rightarrow \text{ at } O(h^6) \text{: if Yukawa for electron much smaller than for muon:} \]

\[ n_{L_\mu} - n_{\bar{L}_\mu} \text{ strongly washed-out} \]
\[ n_{L_e} - n_{\bar{L}_e} \text{ much less washed-out} \]

\[ \Rightarrow \sum_{i=e,\mu,\tau} (n_{L_i} - n_{\bar{L_i}}) = - \sum_{\alpha} (n_{N_\alpha} - n_{\bar{N}_\alpha}) \neq 0 \]

corresponding asymmetry by sphalerons