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Venus: Crater distribution and plains resurfacing models

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Abstract. Detailed analysis of the distribution of craters on Venus using *M*th nearest neighbor analysis, coupled with models based upon surface morphology constraints, indicates that the hypothesis of complete spatial randomness (CSR) cannot be rejected, but is not a unique model of the observed crater distribution. Based on morphologic mapping, the extensive volcanic plains can be divided into four units that have a spread in age of the order of $0.5T$ (the mean surface age of the planet). This four-unit plains model, along with its derivatives, produce test statistics that indicate such models also cannot be rejected. Further, the probability of obtaining a result at least as extreme as the observed test statistic given that the null hypothesis (model corresponds to Venus) is true is lowest for the CSR model. There is no particular reason to pick a CSR model (along with its implications for catastrophic resurfacing) as a constraint on the evolution of Venus, and there are geological reasons to choose the multiage models. We find that we cannot distinguish statistically among models that have two, three, or four distinct production ages within the plains. However, the hypothesis that the variation in crater density within all of the plains is due to a single random process can be rejected for two reasons. First, the binomial probability that such a process could exist within each of the plains units is ≤ 0.05 except the smallest and youngest unit, PL1. Second, using a chi-squared statistic to test the hypothesis that four plains units have the same age gives a p value of 10^{-4}, indicating confident rejection of the hypothesis. Thus CSR cannot be used as a constraint on models of resurfacing or planetary evolution of Venus because of the nonuniqueness in matching such a model to the observed crater distribution and the strong indication of distinct ages within the plains with a significant spread in age. Geological and geophysical constraints provide our best clues for understanding Venus.

1. Introduction

Understanding the nature of how Venus has been resurfaced is essential for gaining insight into the planet's evolution. The debate over Venusian resurfacing has revolved around two core pieces of information: the spatial distribution of impact craters on the surface, and the number of craters embayed from the exterior by lava. Specifically, the nature of the crater distribution, whether it is random or not, has been the key point in all models that described resurfacing.

Previous work concluded that the observed crater distribution could not be distinguished from one that is completely spatially random (CSR) [*Phillips et al.*, 1992; *Strom et al.*, 1994]. The impact cratering process is assumed to be a Poisson random process (craters are as likely to occur in one place as another, and each crater location is completely independent of all other crater locations). This means that if the observed crater distribution is truly CSR, it is necessary to find processes that have operated on the surface during the time of crater emplacement that would leave the distribution in a spatially random state (although not necessarily the original configuration). The simplest way to do this, of course, is by the null process; nothing disturbs the original CSR population. On Earth, the known craters are not randomly distributed [*Koeberl and Anderson*, 1996], and this is a direct consequence of continuing geologic processes such as erosion and plate tectonics. The surface of Venus shows only minor effects of erosion [*Arvidson et al.*, 1992]; therefore the only known ways to remove craters are through volcanism, tectonism, or some combination of the two.

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Out of this previous conclusion of CSR came an extraordinary and widely accepted paradigm for Venusian history, the global (or catastrophic) resurfacing model (CRM) [*Schaber et al.*, 1992; *Strom et al.*, 1994]. CRM implies a rapid, large-scale lava flooding and tectonic event, which would have erased all evidence of earlier craters, followed by a cessation, or at least rapid waning, of resurfacing activity on Venus. Alternatively, a longer period of resurfacing ended rather abruptly in a geologic time frame. CRM is based upon the hypothesis of CSR and an observation of relatively few craters embayed from the exterior by lava [*Phillips et al.*, 1992; *Schaber et al.*, 1992; *Strom et al.*, 1994]. Original work on CRM suggested that the resurfacing tailed off in ~10 Myr [*Schaber et al.*, 1992; *Strom et al.*, 1994]; recent work suggests that the period of waning for this model may have lasted as long as 100 Myr [*Basilevsky et al.*, 1997]. While CRM is certainly the most simple hypothesis, it is not a unique interpretation of the evidence.

Early tests for the hypothesis of CSR, such as the graphical tests of *Phillips et al.* [1992] and the test by *Strom et al.*

[1994] did not provide for a quantifiable level of confidence in the hypothesis of CSR. A more sensitive technique for testing the distribution of point patterns utilizing nearest neighbor distances [e.g., *Hertz*, 1909; *Clark and Evans*, 1954; *Scott and Tout,* 1989; *Squyres et al*., 1997] has been reformulated for the surface of a sphere, a good approximation for the surface of a planet. Hypothesis testing in this analysis is, however, similar to that used by *Phillips et al.* [1992] (in their Table 1, the "object-to-nearest-object distance" is the crater nearest neighbor). Results of analysis using this technique suggest that CSR is not necessarily the best, nor a unique representation of the real crater distribution. This is consistent with the point that the craters are obviously nonrandom with respect to geology in general [*Price and Suppe*, 1994; *Namiki and Solomon*, 1994; *Price and Suppe*, 1995; *Price et al*., 1996], and the plains in particular [*Price*, 1995, Resurfacing history of the Venusian plains, submitted to *Journal of Geophysical Research*, 1998, hereafter referred to as submitted paper]. In addition, it supports the argument against CSR as a constraint on resurfacing and planetary evolution models like CRM.

Resurfacing models integrating constraints from surface geology or morphology should provide greater insight into how Venus has evolved than descriptions involving only spatial relationships of craters. Utilizing morphologic mapping of the planet [*Price and Suppe*, 1995; *Price*, 1995, submitted paper, 1998] and, in particular, the plains units, which comprise more than 60% of the surface, we have constructed several models (in addition to a single-age model for the plains) in an attempt to explain resurfacing. The plains are morphologically delineated into four units (PL1, PL2, PL3, and PS), corresponding to decreasing flow lobateness from PL1 to PS [*Price*, 1995, submitted paper, 1998; *Tanaka et al*., 1997].

In this paper we use statistical procedures to reaffirm that the hypothesis of a CSR distribution of craters cannot be rejected. But we show that non-CSR models based on geomorphic mapping of plains units, and with a spread in plains age of approximately one-half the mean age of the surface or more, also cannot be rejected. Thus CSR is not a unique constraint for studying the evolution of Venus.

2. Randomness?

The first step in analyzing the impact crater distribution is to test the null hypothesis that the distribution illustrates complete spatial randomness. For testing statistical hypotheses on spatial distributions, distance-based methods [*Diggle*, 1983] are sometimes more sensitive than other methods such as quadrant counting, where the size of the areas chosen can affect the conclusion. Indeed, the P-P and Q-Q plots [*Diggle*, 1983] used previously [*Phillips et al*., 1992] are an example of distance-based methods. Unfortunately, they are not sufficiently sensitive to distinguish among strongly different resurfacing models, as illustrated in Figures 1 and 2 of *Hauck et al.* [1997]. The main problem with P-P and Q-Q plots is that they distinguish only among the most extreme possibilities under which a stochastic process could be present. *Strom et al.* [1994] tested for randomness using a chi-squared test on sine of latitude. *Kirk et al.* [1995] noted that this test done by *Strom et al.* [1994] is less sensitive than methods used by *Phillips et al.* [1992]. This indicates that the debate on resurfacing has centered around conclusions based on the tests of *Phillips et al.* [1992]. However, this debate should properly have focused on the p values in Table 1 of *Phillips et al.* [1992]. The null hypothesis (H_0) is that the crater distribution is CSR. The p value is the probability that a test statistic ("value" in their Table 1) is at least as extreme as observed given that H_0 is true. A customary threshold value (α, the level of significance) for the p value is 0.05 (or less), or a 1 in 20 chance that such an outcome would occur if H_0 is true. Values of p less than 0.05 would then lead to a rejection of H_0. The smallest p value, among various test statistics, for craters in this table is 0.30, while for coronae many p values are 0.01. Thus H_0 cannot be rejected for craters but it can (as is obvious from their spatial distribution) for coronae [*Phillips et al*., 1992]. This does not mean, however, that the crater distribution is necessarily CSR. In order to prove that, H_0 would have to be rejected for all other (plausible) models of the crater distribution.

Here we use the mean distance between the first nearest (crater) neighbor, second nearest neighbor, etc., as test statistics for the null hypothesis. The mean value under "object-to-nearest-object distance" in Table 1 of *Phillips et al.* [1992] is in fact the equivalent to what we do here for (first) nearest neighbor tests of CSR. Because of large data gaps in Magellan images, a rank test was used to arrive at approximate p values for test statistics of *Phillips et al*. [1992]. Because data are available for almost the entire sphere now, analytic and quasi-analytic expressions can be used to arrive at p values. A method of testing the mean distance between nearest neighbors on a plane [*Clark and Evans*, 1954], used recently in studying crater saturation on Rhea and Callisto [*Squyres et al*., 1997], yields a test statistic and a p value for CSR that is appropriate for areas whose linear dimensions are small compared to the radius of a given planet. We have re-derived the nearest neighbor test statistics of *Clark and Evans* [1954] for the surface of a sphere (see appendix). It turns out that an independent formulation [*Scott and Tout*, 1989] of the same process for the distribution of astronomical objects yields the same final formulation for testing the distribution of points on a sphere in the asymptotic limit (large number of objects). More important, this independent formulation also results in test statistics for the mean distance to 2nd, 3rd, ..., Mth nearest neighbors. By looking at the distance to the first few nearest neighbors (i.e., 1-4, following the precedent of *Scott and Tout* [1989]), we can test the null hypothesis over a wider range of length scales than the first nearest neighbor alone, without degrading the sensitivity of the test statistic.

Table 1 gives two-sided p values for the first through fourth nearest neighbors for testing the null hypothesis that that the Venusian crater distribution and simulations of CSR

Table 1. Two-Sided p Values for Testing the Null Hypothesis of Poisson Randomness for the Mth Nearest Neighbor

M	All Craters	Subset	Simulations
1	0.49	0.34	0.95
2	0.18	0.07	0.85
3	0.31	0.19	0.89
4	0.31	0.18	0.91

Column labeled "all craters" is for the total observed population of 939 craters. The subset column refers to 926 craters that match the morphologic mapping of the planet from 82.5°N to 82.5°S. The simulations column gives results of 201 simulations of homogeneously random cratering, an approximation of CSR.

are indeed random. The p values for all 939 craters provide the best clues as to how the craters are distributed. In particular, the p values range from 0.18 to 0.49, so that H_0 certainly cannot be rejected at the traditional α level of 0.05. Departing from statistical custom, we can state that there is a probability of only 0.5 or less of obtaining the test statistics (equation (A20) in the appendix; values not shown in Table 1) given that H_0 is true. The test of a purely homogeneous random cratering model (simulations column) shows that the probability approaches unity of obtaining the test statistics for M = 1-4, given the hypothesis that the numerical simulation of randomness (using pseudorandom number generation and three-axis sampling from a uniform distribution) has a CSR distribution. This is based on the mean distance to the Mth nearest neighbor from 201 random simulations, and demonstrates the quality of spatially random simulations here.

Besides testing the null hypothesis, we can calculate the ratio of the mean intercrater distance to the expected Poisson value and obtain a measure of the direction of deviation from randomness. If this ratio is equal to 1, that would indicate a random population; values greater than 1 indicate a tendency toward uniformity, and less than 1 indicate a tendency toward clustering [*Clark and Evans*, 1954]. For all nearest neighbors in the real population of 939 craters the ratio values are less than 1, indicating clustering. If the clustering is nonrandom, it could be the result of two types of processes: (1) objects have a tendency to form near each other, and (2) a process is removing objects from a randomly emplaced field such that clusters are created by the absence of objects in certain areas. Resurfacing is most likely a process that is spatiotemporally nonrandom. It is possible that the crater removal processes have a realm of influence of the order of the size of the Mth nearest neighbor distances tested (i.e., less than ~1000 km is the scale on average). It is unlikely that the impact craters' formation is clustered (we have counted multiple craters from the same impact as a single crater in the same manner as the database of *Herrick et al.* [1997]). Therefore, it is likely that resurfacing processes have operated over the time span of crater emplacement.

3. Morphology and Plains Ages

Morphologic mapping (from Magellan synthetic aperture radar (SAR) images) based upon degree of dominant "lobateness" of lava flows and their characteristic radar brightness reveals that units of distinctly different crater production ages exist on the surface of Venus [*Price*, 1995, submitted paper, 1998; *Tanaka et al.*, 1997]. Specifically, the hypothesis is that the number of recognizable lobate lava flow fronts in a region decrease with increasing surface age, and a prediction of this hypothesis is that there is a regional anticorrelation between lobateness and crater density. The fourfold morphologic division of plains units, from youngest to oldest, are termed PL1, PL2, PL3, and PS. They cover more than 60% of the planet and are thus the most significant features from a resurfacing standpoint. Other studies [*Price and Suppe*, 1994; *Namiki and Solomon*, 1994; *Price et al.*, 1996] have used impact crater densities in this way (defining units based on geology or geomorphology and then examining their crater densities) to constrain the ages of rifting and volcanism on Venus. In radar data it is the roughness of the surface that is a determining factor in radar brightness; it is a difference between the flow roughness and background units that determines their visibility. For this reason, if a flow were to erupt onto a surface that had a roughness similar to previously emplaced flows, it would be difficult to determine the lobateness of the flow. In addition, it is important to remember that this definition of the plains units is based upon morphologic rather than stratigraphic criteria, and the actual production ages may overlap between units. Even with these caveats, the different production ages imply that, on average, surface ages increase as lobateness and radar brightness decrease. Other workers [*Basilevsky et al.*, 1996] have, to a first-order, compared the stratigraphic units of *Basilevsky and Head* [1995] with the morphologic plains units used here and concluded that while the morphologic units capture the overall stratigraphic progression, they cannot distinguish fine-scale details. It is also important to note that lack of resolution of the geomorphic indexing precludes the mapping of subunits within each of the four plains units; this may be particularly significant in the oldest unit, PS, due to the diffuse nature of the flow boundaries. We note, however, that since each morphologic unit represents on average one or more time-stratigraphic units, the time span of plains emplacement (either continuous or discrete events spread in time) must equal or exceed the difference in age between the youngest and oldest morphologic unit.

The average apparent production age (T) of Venus' surface had been estimated as approximately 300 Ma [*Schaber et al.*, 1992] and 500 Ma [*Phillips et al.*, 1992]. Recent work [*McKinnon et al.*, 1997] has revised this age estimate to about 700-800 Ma. Using a database of 926 impact craters that match the plains and all other areas of the planet mapped by morphology [*Price and Suppe*, 1995; *Price*, 1995, submitted paper, 1998], we calibrated the relative ages of the plains units using 750 Ma as the global mean surface age [*McKinnon et al.*, 1997] (Table 2). The age for the plains if they were all a single unit would be 829 ± 65 Ma. This is close to the age of PL3, and it overlaps, within mutual 2σ error bars, all of the units except PL2. The 2σ values are calculated using two standard deviations of a Poisson approximation to the binomial distribution as discussed by *Price et al.* [1996]. We emphasize that these morphologic units have been defined independently of crater density. Because PL2 and a grouping of the younger plains as PL1+PL2 have ages distinct (outside of 2σ) from a single-age plains model (see Table 2), it follows that there must be real geologic units within the plains with significantly different ages. This can be demonstrated more quantitatively in two ways. First, we can examine the probability that the number of craters is a specific unit is expected from a spatially random global as has been done by *Price et al.* [1996] for rifting and volcanism. What we observe from our results (Table 2, column labeled P) is that we can reject at the standard level of 0.05 the hypothesis that individually any of the three oldest plains units could occur in a random sample with the global mean crater density. Because of the small size and few craters in PL1, it is not unexpected that it cannot be rejected that the number of craters could occur in a random population using this technique. However, it is important to note that groupings of PL1+PL2 and PL3+PS can be rejected as being from a random population on the basis of binomial probabilities. Second, a chi-squared test is well suited to test how well the hypothesis of a single age for the plains fits with the observed distribution of craters within the plains units. This can be accomplished by comparing the observed number of craters in each unit with the expected number for each unit if the plains are a single age. We find a p value of 10^{-4} for this test, indicating that we can reject the hypothesis that a

Table 2. Relative and Absolute Ages of Venusian Plains Units

Unit	Area	Craters	P^a	Relative Age	Estimated Age (Ma)
PL1	11.86	19	1.7×10^{-1}	$0.79 \pm 0.36\ T$	589 ± 270
PL2	84.43	149	3.0×10^{-2}	$0.87 \pm 0.14\ T$	649 ± 106
PL3	93.05	217	1.3×10^{-2}	$1.14 \pm 0.16\ T$	857 ± 116
PS	99.82	267	4.5×10^{-7}	$1.31 \pm 0.16\ T$	983 ± 120
PL1+PL2	96.29	168	1.4×10^{-2}	$0.86 \pm 0.13\ T$	641 ± 99
PL3+PS	192.87	484	6.5×10^{-10}	$1.23 \pm 0.11\ T$	923 ± 84
SAP	289.16	652	4.2×10^{-6}	$1.11 \pm 0.09\ T$	829 ± 65

A unit of area is 10^6 km^2. Errors listed are 2σ. Note that both PL2 and PL1+PL2 have relative ages that do not overlap within 2σ of the single-age plains (SAP) model, suggesting that the younger plains have distinct ages that are statistically significant. The mean surface production age, used to calculate the last column, is estimated as $T = 750$ Ma [*McKinnon et al.*, 1997].

[a] Probability of finding a number at least as extreme as the terrain number of craters occurring in a random sample of a spatially random population with the global population.

single age for the plains is consistent with the observed variations in crater density across the plains units. However, when we extend this test to the two- and three-age models versus the observed variations in crater density, we get p values of 0.5, indicating that we cannot reject the hypothesis that the plains are consistent with two or three subunits, as well the original four subunits.

It is apparent from these results that, even if the plains' crater distribution is CSR, the crater counting statistics suggest that resurfacing may have extended over a considerable period of time, perhaps of the order of 0.5 Gyr. Of course, none of these models (CRM, 4 plains units, etc.) provide insight into how far back into the past that resurfacing might have extended. In any event, the spread in ages of the observable plains is likely at least half the average surface age of the planet, irrespective of the absolute average age of the surface. This suggests another scenario for the resurfacing history of Venus that is quite different from CRM.

4. Models

An alternative to CRM that we can test is the end-member hypothesis that four large resurfacing events, corresponding to the four morphologic units, were responsible for creating the plains. (The other end-member is that the four units sample a continuum of plains emplacement at a constant rate.) The four-event model may represent only a crude representation of plains emplacement, but we are only asking a simple question: Does this model provide a better match to the actual crater distribution than CRM? Using Monte Carlo techniques, we tested the hypothesis that Venus could have been resurfaced over four widely spread periods ("nominal" model). While craters were generated from a spatially random probability sampling for the entire Venus sphere, their accumulation on the planet was constrained by surface morphology. This was accomplished by assuming that each mapped unit on the planet (including PL1, Pl2, Pl3, PS) represented a single crater production age. For any unit, we stopped counting craters when the number of "hits" equaled the actual number of craters observed. This is equivalent to starting "hits" later in the simulation to be consistent with the production age of that unit. When a simulation was finished, we calculated statistics on the crater distribution of the entire sphere. The result is a planetary surface with the same number of craters as are observed, and in the same proportions by morphologic unit (hence they have the 400 Myr spread in production age given in Table 2).

In order to examine the sensitivity of this method, we intentionally perturbed the relative ages of two plains units and ran our models again. Continuity among the models was maintained by requiring that the total number of craters was the same in all models. First, the age of PS was increased by 2σ and PL3 was decreased by 2σ ("MB1" model). In the second test ("MB2" model), PL2 was decreased and PS was increased. The intent was to examine the effect of more disparate unit ages, within the 2σ limits, on the model hypothesis of four distinct global events.

In addition to these sensitivity tests, we examined three other models that were designed to analyze whether fewer subdivisions of the plains might be more representative than the nominal model. The results were two dual-age plains ("DAP" and "DAP2") that examined whether the two younger plains units (PL1+PL2) and two older plains units (PL3+PS) should be grouped together, and a tri-age plains model ("TAP") that tested whether just the two oldest units should be grouped together. Both DAP and TAP represent models based upon mean plains ages, whereas DAP2 is a sensitivity model like MB1 and MB2, where the older plains age was increased and the younger plains decreased within 2σ limits. Table 3 details the actual ages (as a fraction of the mean surface age of the planet T) that were used in each of the models described here.

The age-based models use a subset of craters (926) that corresponds to the morphologic database used. They have consistently lower two-sided p values with respect to the null hypothesis of randomness than the complete set of craters (939 craters; see Table 1). This is an obvious result, given that the mean and standard error of theoretical distribution is based on the entire sphere, so that "missing" craters drive the observed distribution toward nonrandomness as long as the excluded craters themselves are spatially nonrandom. Craters are missing from the database because the mapping [*Price and Suppe*, 1995; *Price*, 1995; *Price et al.*, 1996] extends from 82.5°S to 82.5°N and the pole-dwelling craters are pref-

Table 3. Relative Ages (as a Fraction of *T*) of Venusian Plains Units Used in Each of the Resurfacing Models Tested

Model	PL1	PL2	PL3	PS
Nominal	0.79	0.87	1.14	1.31
MB1	0.79	0.87	0.98	1.47
MB2	0.79	0.73	1.14	1.43
SAP	1.11	1.11	1.11	1.11
DAP	0.86	0.86	1.23	1.23
DAP2	0.73	0.73	1.29	1.29
TAP	0.79	0.87	1.23	1.23

Model designations are discussed and defined in the text.

erentially and nonrandomly missing in the database. In the resurfacing models discussed in this section, the theoretical mean and standard error of a CSR model are replaced with numerically determined values from 200 Monte Carlo simulations that count "hits" only in the mapped area. The test statistic (equation (A20)) is calculated from these values, and the observed *M*th nearest neighbor mean determined from the 926 craters in the database. There is no loss in the utility of these calculations when performed in this manner.

Figure 1 illustrates the two-sided p values for the null hypotheses that one of the multiage models, the single-age plains, or CRM (a CSR model) is representative of Venus. An obvious feature of these results is the characteristically lower p values for the second nearest neighbor. This is a property of points distributed on the surface of sphere that is evident whether one is testing hypotheses versus real data, numerical simulations, or analytical theory. For example, this dip at $M = 2$ occurs in the p values for "CSR versus Random," simulations of random points on a sphere versus the Poisson random theory (see appendix). Figure 1 shows H_0 certainly cannot be rejected at the traditional α level of 0.05 for any of the models. Conservatively, there is no particular reason to pick CSR (and the CRM implication) as a constraint on the evolution of Venus, but there are geological reasons to pick the multiage models. We also note that the probability of obtaining results as extreme as the observed test statistic given that H_0 is true is lowest for the CSR model. We also observe that the p values of the nominal, DAP, and TAP models are so close to each other that we are unable to distinguish among models with two or more plains ages, which is consistent with our observations from chi-squared tests on the terrain densities of these models.

5. Discussion and Conclusions

*M*th nearest neighbor analysis is a technique that allows exploration of the relationships among all the craters within a distribution. Such analysis of the Venusian impact crater distribution, coupled with models based upon geologic criteria, enforce a view of Venus quite different from the current paradigm: interpretations of the crater distribution on Venus are nonunique, and models that incorporate geological constraints are statistically at least as likely as CSR. Therefore CSR should not be taken as a rigid constraint when describing the resurfacing history of Venus [*Phillips et al.*, 1992; *Schaber et al.*, 1992; *Bullock et al.*, 1993; *Strom et al.*, 1994].

The mean distance between nearest neighbors and the p values related to randomness may provide clues as to possible styles of resurfacing that should be investigated further. The mean surface distance between first nearest crater neighbors is of the order of 350 km, and 780 km for the fourth nearest. We can also compare the mean distance to the first nearest neighbor within each of the young (PL1+PL2) and old (PL3+PS) plains units to get an idea of the degree of potential clustering among and within the units. What is observed is that both the young and old plains have the same mean distance to first nearest neighbor, about 334 km. This is a somewhat surprising result, in that one would expect that a unit with a higher density of craters, such as the old plains, should have a distance to the nearest crater that is smaller than that of a unit with a lower density, such as the young plains. However, this can be explained conceptually if the young plains actually contain portions of older plains in such a way

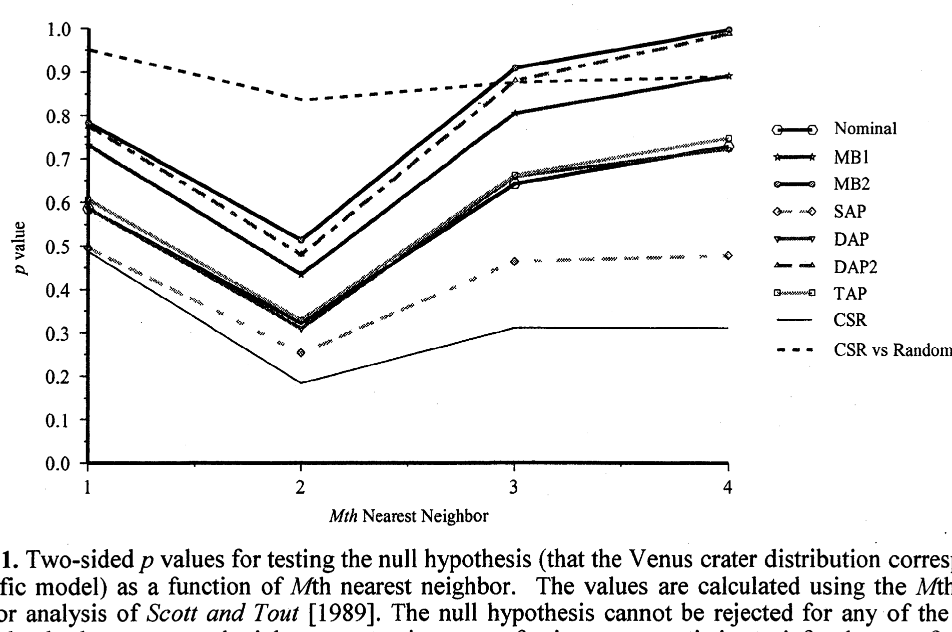

Figure 1. Two-sided p values for testing the null hypothesis (that the Venus crater distribution corresponds to a specific model) as a function of *M*th nearest neighbor. The values are calculated using the *M*th nearest neighbor analysis of *Scott and Tout* [1989]. The null hypothesis cannot be rejected for any of the models, which clearly demonstrates the inherent nonuniqueness of using crater statistics to infer the resurfacing history.

that small "clusters" are preserved, so that the distance to the first nearest neighbor is the same in both units. This suggests a very "patchy" style to recent resurfacing, with inliers of older plains preserved within regions of younger plains. Further, this underscores the regional averaging that must take place when defining the four morphologic units.

In order to understand the evolution of Venus it is necessary to look beyond simple mathematical descriptions of a sparse crater distribution. Geologic constraints, such as surface morphology, provide greater physical insight and better models of how Venus has resurfaced. It is indeed evident that the overall formation of the plains has the most significant influence on the distribution of craters on Venus. We have not addressed how an embayed crater constraint might be best employed, but we have considered the more fundamental and quantifiable issue of the distribution of craters and plains resurfacing.

Several key points suggest that a shift away from the current paradigm of CRM is necessary. First, mapping of the surface based upon morphologic constraints indicates that there is a spread in ages ($\geq$400 Myr) in the volcanic plains that is larger than half of the mean production age of the surface. Second, there is a significant (more than 2σ deviation) separation in ages between the single-age plains model and PL2, indicating that there are real, distinct units within the plains of significantly different ages. Third, independent statistical tests using binomial probabilities and chi-squared tests on the number of craters within the plains units suggest that the distinct ages are real and that a single-age interpretation is inconsistent with the crater density variations within the plains. Finally, it cannot be emphasized strongly enough that interpretation of the crater distribution is inherently nonunique, requiring a reliance on geological and geophysical data to constrain the resurfacing and planetary evolution of Venus.

This leaves us with the question of how these conclusions affect geodynamical models that attempt to explain Venus based on the hypothesis of CRM [i.e., *Parmentier and Hess*, 1992; *Turcotte* 1993, 1995; *Arkani-Hamed et al.*, 1993]. In light of our conclusions regarding the nonuniqueness of resurfacing models and the important role played by geological constraints, it is obvious that CRM is neither necessary nor sufficient to explain the evolution of Venus. In addition, CSR cannot be used either as a unique constraint upon these geodynamical models or as a premise to an hypothesis to formulate such models.

Appendix

We have re-derived for a sphere the formulas for the expected mean distance and standard error to nearest neighbors of a random population on a plane [*Clark and Evans*, 1954]. We start with the Poisson probability density function

$$p(x)=\frac{m^x e^{-m}}{x!} \tag{A1}$$

where x is the random variable (probability that x craters will be found) and m is the expected number of craters in an area. Because we are interested in the surface of a sphere, we use the area of a spherical cap to find m:

$$m=2\rho\pi R^2(1-\cos\theta)/k \tag{A2}$$

where ρ is the global mean density, R is the radius of the planet, θ is the angular radius of the cap, and k is the number of sectors into which the cap is divided. The probability that no craters will be found in the sector is then

$$p(0)=e^{-m}=e^{-2\rho\pi R^2(1-\cos\theta)/k} \tag{A3}$$

which is the same as the probability that the nearest neighbor is a distance θ away from the central point. Note that θ is now the random variable. We use the cumulative density function to find the probability density function. It can only be defined over the interval (0, π) and is given by

$$F(\theta)=\frac{1-e^{-2\rho\pi R^2(1-\cos\theta)/k}}{1-e^{-4\rho\pi R^2/k}} \tag{A4}$$

The probability density function is obtained from

$$p(\theta)=\frac{dF(\theta)}{d\theta} \tag{A5}$$

so that using (A4) we obtain

$$p(\theta)=\frac{\left(2\rho\pi R^2/k\right)\sin\theta\, e^{-2\rho\pi R^2(1-\cos\theta)/k}}{1-e^{-4\rho\pi R^2/k}} \tag{A6}$$

Now we find the first and second moments of the probability density function, $E(\theta)$ and $E(\theta^2)$, respectively:

$$E(\theta)=\int_0^{\pi} p(\theta)\theta\, d\theta$$

$$=\int_0^{\pi}\frac{\left(2\rho\pi R^2/k\right)\sin\theta\, e^{-2\rho\pi R^2(1-\cos\theta)/k}}{1-e^{-4\rho\pi R^2/k}}\theta\, d\theta$$

$$=\frac{\left(2\rho\pi R^2/k\right)e^{-2\rho\pi R^2/k}}{1-e^{-4\rho\pi R^2/k}}\int_0^{\pi} e^{2\rho\pi R^2\cos\theta/k}\theta\sin\theta\, d\theta \tag{A7}$$

Defining α as

$$\alpha\equiv 2\rho\pi R^2/k \tag{A8}$$

then

$$E(\theta)=\frac{\alpha e^{-\alpha}}{1-e^{-2\alpha}}\int_0^{\pi} e^{\alpha\cos\theta}\theta\sin\theta\, d\theta$$

$$=\frac{\alpha}{2\sinh\alpha}\int_0^{\pi} e^{\alpha\cos\theta}\theta\sin\theta\, d\theta \tag{A9}$$

This can be shown to integrate to

$$E(\theta)=\frac{\pi}{2\sinh\alpha}\left[I_0(\alpha)-e^{-\alpha}\right] \tag{A10}$$

where I_0 is a modified Bessel function of zero order:

$$I_0(\alpha) = \frac{1}{\pi}\int_0^{\pi} e^{\alpha\cos\theta}\, d\theta \tag{A11}$$

When $k = 1$ and N is the number of craters on the sphere,

$$\alpha = 2\rho\pi R^2 = \frac{N}{4\pi R^2}2\pi R^2 = \frac{N}{2} \tag{A12}$$

For Venus, $N = 939$ so $\alpha = 469.5$, so asymptotic forms are appropriate in (A10):

$$I_0(\alpha) \approx \frac{e^{\alpha}}{\sqrt{2\pi\alpha}} \tag{A13}$$

$$\sinh(\alpha) \approx \frac{e^{\alpha}}{2} \tag{A14}$$

Therefore, $E(\theta)$ simplifies to

$$E(\theta) = \sqrt{\frac{\pi}{N}} \tag{A15}$$

The first moment is the mean or expected distance between nearest neighbor craters on a sphere:

$$\bar{\theta}_E \equiv E(\theta) = \sqrt{\frac{\pi}{N}} \tag{A16}$$

The second moment $E(\theta^2)$ is

$$E(\theta^2) = \frac{\alpha e^{-a}}{1-e^{-2\alpha}}\int_0^{\pi} e^{\alpha\cos\theta}\theta^2 \sin\theta\, d\theta \tag{A17}$$

A simple, analytic solution to this problem has not been found, but it can be shown to reach numerically the following asymptotic limit when $\alpha = N/2$ is large:

$$E(\theta^2) = \frac{4}{N} \tag{A18}$$

From here is it a simple exercise to calculate the standard error:

$$\sigma_E = \sqrt{\frac{4-\pi}{N^2}} \tag{A19}$$

Both the mean and standard error have forms that are similar to the planar case but are generalized to the surface of a sphere. We use (A16) and (A19) to obtain the standard normal variable, which becomes the test statistic for calculating two-sided p values for testing the null hypothesis:

$$z = \frac{\bar{\theta}_{obs} - \bar{\theta}_{exp}}{\sigma_E} \tag{A20}$$

Scott and Tout [1989] follow a similar procedure for deriving expressions (A15) and (A18), but they do it from a probability density function that has been derived for the *M*th nearest object :

$$P_M(\theta) = \frac{(N-1)!}{2^{N-1}(N-M-1)!(M-1)!}\sin\theta \times (1-\cos\theta)^{M-1}(1+\cos\theta)^{N-M-1} \tag{A21}$$

which allows for the *M*th nearest neighbor analysis in the main text.

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References

Arkani-Hamed, J., G. G. Schaber, and R. G. Strom, Constraints on the thermal evolution of Venus inferred from Magellan data, *J. Geophys. Res.*, *98*, 5309-5315, 1993.

Arvidson, R. E., R. Greeley, M. C. Malin, R. S. Saunders, N. Izenberg, J. J. Plaut, E. R. Stofan, and M. K. Shepard, Surface modification of Venus as inferred from Magellan observations of plains, *J. Geophys. Res.*, *97*, 13,303-13,318, 1992.

Basilevsky, A. T., and J. W. Head, Global stratigraphy of Venus: Analysis of thirty-six test areas, *Earth Moon Planets*, *66*, 285-336, 1995.

Basilevsky, A. T., G. A. Burba, and V. P. Kryuchkov, Stratigraphy of Venusian plains: Comparison of Price [1995] and Basilevsky and Head [1995] units (abstract), *Lunar Planet. Sci. Conf.*, *XXVII*, 73-74, 1996.

Basilevsky, A. T., J. W. Head, G. G. Schaber, and R. G. Strom, The resurfacing history of Venus, in *Venus II*, edited by S. W. Bougher, D. M. Hunten, and R. J. Phillips, pp. 1047-1086, Univ. of Ariz. Press, Tucson, 1997.

Bullock, M. A., D. H. Grinspoon, and J. W. Head, Venus resurfacing rates: Constraints provided by 3-D Monte Carlo simulations, *Geophys. Res. Lett.*, *20*, 2147-2150, 1993.

Clark, P. J., and F. C. Evans, Distance to nearest neighbor as a measure of spatial relationships in populations, *Ecology*, *35*, 445-453, 1954.

Diggle, P. J., *Statistical Analysis of Spatial Point Patterns*, Academic, San Diego, Calif., 1983.

Hauck, S. A., II, R. J. Phillips, and M. H. Price, Implications for Venusian plains resurfacing from geomorphic mapping and impact crater densities (abstract), *Lunar Planet. Sci. Conf.*, *XXVIII*, 527-528, 1997.

Herrick, R. R., V. L. Sharpton, M. C. Malin, S. N. Lyons, and K. Feely, Morphology and morphometry of impact craters, in *Venus II*, edited by S. W. Bougher, D. M. Hunten, and R. J. Phillips, pp. 1015-1046, Univ. of Ariz. Press, Tucson, 1997.

Hertz, P., Über den geigenseitigen durchschnittlichen Abstand von Punkten, die mit bekannter mittlerer Ditchte im Raume angeordnet sind, *Math. Ann.*, *67*, 387-398, 1909.

Kirk, R. L., G. G. Schaber, R. G. Strom, New statistical results on the spatial distribution and physical properties of impact craters on Venus (abstract), *Lunar Planet. Sci. Conf.*, *XXVI*, 757-758, 1995.

Koeberl, C., and R. R. Anderson, Manson and company, impact structures in the United States, in *The Manson Impact Structure, Iowa: Anatomy of an Impact Crater*, edited by C. Koeberl and R. R. Anderson, *Spec. Pap Geol. Soc. Am.*, *302*, 1-29, 1996.

McKinnon, W. B., K. J. Zahnle, B. A. Ivanov, and H. J. Melosh, Cratering on Venus: Models and observations, in *Venus II*, edited by S. W. Bougher, D. M. Hunten, and R. J. Phillips, pp. 969-1014, Univ. of Ariz. Press, Tucson, 1997.

Namiki, N., and S. C. Solomon, Impact crater densities on volcanoes and coronae on Venus: Implications for volcanic resurfacing, *Science*, *265*, 929-933, 1994.

Parmentier, E. M., and P. C. Hess, Chemical differentiation of a convecting interior: Consequences for a one plate planet such as Venus, *Geophys. Res. Lett.*, *19*, 2015-2018, 1992.

Phillips, R. J., R. F. Raubertas, R. E. Arvidson, I. C. Sarkar, R. R. Herrick, N. Izenberg, and R. E. Grimm, Impact craters and Venus resurfacing history, *J. Geophys., Res.* *97*, 15,923-15,948, 1992.

Price, M. H., Resurfacing history of the Venusian plains based on the distribution of impact craters (abstract), *Lunar Planet. Sci. Conf.*, *XXVI*, 1142-1143, 1995.

Price, M. H., and J. Suppe, Mean age of rifting and volcanism on Venus deduced from impact crater densities, *Nature*, *372*, 756-759, 1994.

Price, M. H., and J. Suppe, Constraints on the resurfacing history of Venus from the hypsometry and distribution of volcanism, tectonism, and impact craters, *Earth Moon Planets*, *71*, 99-145, 1995.

Price, M. H., G. Watson, J. Suppe, and C. Brankman, Dating volcanism and rifting using impact crater densities, *J. Geophys. Res.*, *101*, 4657-4671, 1996.

Schaber, G. G., R. G. Strom, H. J. Moore, L. A. Soderblum, R. L. Kirk, D. J. Chadwick, D. D. Dawson, L. R. Gaddis, J. M. Boyce, and J. Russell, Geology and distribution of impact craters on Venus: What are they telling us?, *J. Geophys. Res.*, *97*, 13,257-13,301, 1992.

Scott, D., and C. A. Tout, Nearest neighbor analysis of random distributions on a sphere, *Mon. Not. R. Astron. Soc.*, *241*, 109-117, 1989.

Squyres, S. W., C. Howell, M. C. Liu, and J. J. Lissauer, Investigation of crater "saturation" using spatial statistics, *Icarus*, *125*, 67-82, 1997.

Strom, R. G., G. G. Schaber, and D. D. Dawson, The global resurfacing of Venus, *J. Geophys. Res.*, *99*, 10,899-10,926, 1994.

Tanaka, K. L., D. A. Senske, M. Price, and R. L. Kirk, Physiography, geomorphic/geologic mapping, and stratigraphy of Venus, in *Venus II*, edited by S. W. Bougher, D. M. Hunten, and R. J. Phillips, pp. 667-694, Univ. of Ariz. Press, Tucson, 1997.

Turcotte, D. L., An episodic hypothesis for Venusian tectonics, *J. Geophys. Res.*, *98*, 17,061-17,068, 1993.

Turcotte, D. L., How does Venus lose heat?, *J. Geophys. Res.*, *100*, 16,931-16,940, 1995.

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