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Variable conductivity: Effects on the thermal structure of subducting slabs

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Abstract. Understanding the thermal structure of the lithosphere in subduction zone environments is a key element in the realization of the source of deep focus earthquakes and other geodynamic processes. Model predictions of the thermal structure of subducting slabs are dependent upon *a priori* knowledge of how thermal conductivity varies within the slab. Using recent results for the variation of thermal conductivity as a function of temperature, pressure, and gross mineralogy, we investigate how models that utilize a realistic model for variable conductivity compare with canonical models for constant conductivity. Results suggest that jumps in thermal conductivity across the phase boundaries that arise due to transformations to β and γ-spinel phases lead to old, fast, steeply dipping slabs being considerably warmer than previously thought. Although these results do not rule out the possibility of a metastable wedge source of deep focus earthquakes, they do imply the likelihood of metastable olivine being found at depth is less than once thought.

1. Introduction

The thermal structure of subducting lithosphere is key to understanding many geodynamic processes. As such, it has received considerable attention since the acceptance of the plate tectonics paradigm [e.g., *McKenzie*, 1969; *Minear and Toksöz*, 1970; *Hsui and Toksöz*, 1979, *Davies and Stevenson*, 1992, *Daessler et al.*, 1996, *Devaux et al.*, 1997]. In terms of global dynamics, subduction is the primary mode through which the Earth cools itself via the warming of cold slabs within the hotter mantle. Temperature (along with pressure and composition) is also a parameter of central importance in determining the mineralogy and rheology of the lithosphere and mantle, which in turn play a significant role in plate motion and mantle dynamics [i.e. *Riedel and Karato*, 1997; *Marton et al.*, 1999].

Also of some practical interest is the drive to understand the nature and origin of deep focus earthquakes [*Green and Houston*, 1995; *Kirby et al.*, 1996]. Several workers have investigated the possibility that metastable olivine persists below the equilibrium α → β olivine phase boundary at approximately 410 km depth and its potential as the source of deep earthquakes [i.e. *Sung and Burns*, 1976; *Rubie and Ross*, 1994; *Daessler et al.*, 1996; *Devaux et al.*, 1997]. In addition, metastable wedges have implications for the stress field in slabs [*Bina*, 1996; *Goto et al.*, 1997] and negative buoyancy effects on the plate driving force [*Marton et al.*, 1999]. Studies of this type often couple a model of the kinetics of the olivine to spinel phase transition with a thermal model of a subducting slab [*Daessler and Yuen*, 1993, 1996; *Daessler et al.*, 1996; *Devaux et al.*, 1997].

It is obvious that most of the important intrinsic properties that control the Earth's dynamics are not simply constants, but are dependent upon, or interdependent with, several physical properties of the system and its constituents such as temperature, pressure, composition, and mineralogy. Experimental data for the variation of thermal conductivity (k) and thermal diffusivity ($\kappa = k/\rho C_P$ where ρ is the density and C_P is the heat capacity at constant pressure) of mantle materials with temperature have existed for more than three decades [i.e. *Kanamori et al.*, 1968; *Schatz and Simmons*, 1972]. However, conductivity has been assumed to be constant in virtually all models of heat transport within the Earth [i.e. *McKenzie*, 1969; *Minear and Toksöz*, 1970; *Stein and Stein*, 1992; *Daessler et al.*, 1996, *Devaux et al.*, 1997].

Recently, *Hofmeister* [1999] has developed a theoretical framework that describes the substantial variation of thermal conductivity of mantle minerals as a function of temperature and pressure. The increase of k with pressure opposes the decrease with temperature but not enough to compensate for it. In addition, phase changes alter thermal conductivity. Thus, for slabs it is important to incorporate the functionality of k(P, T, mineralogy) in thermal models.

2. Modeling

The transfer of heat within subducting lithosphere can be described simply by the advection-diffusion equation, which in steady state has an analytic solution for simple boundary conditions [*McKenzie*, 1969]. We introduce variable conductivity into the advection-diffusion equation to explore its effects on slab thermal structure. A steady state solution will be sufficient here to describe the first-order effects of heat transfer in the system. We solve the non-linear form of the equation for conservation of energy

$$\rho C_P \mathbf{v} \cdot \nabla T = \nabla \cdot \left[k(T,P,X) \nabla T \right] \qquad (1)$$

where ρ is the density, C_P is the heat capacity at constant pressure, k is the thermal conductivity, $\mathbf{v}$ is the velocity of the slab, P is pressure, X is mineralogy, and T is temperature. We recognize that the use of potential temperature would be more rigorous, but considering the uncertainties in the parameters and the simplifications in the model, the effects of this omission should not seriously alter our conclusions [e.g. *Davies and Stevenson*, 1992]. The effects of thermal coupling between the mantle and slab and flow induced in the mantle by the slab are not included here because we are only

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attempting to assess the importance of variable conductivity, not produce the most realistic thermal model.

Equation (1) is well suited for solution via the finite element method [i.e. *Reddy and Gartling,* 1994]. We developed our own code based upon the Galerkin method and the subroutines of *Braun and Sambridge* [1995] to solve this equation in two dimensions. The code was successfully benchmarked against the analytic solution of *McKenzie* [1969]. Due to the non-linear nature of (1), the Picard (subsequent substitution) iterative method [e.g. *Reddy and Gartling,* 1994] is used to return solutions for both the thermal conductivity and temperature.

We solve equation (1) only in down going portion of the lithosphere. The system is modeled with a constant, finite thickness, length, dip, and down dip velocity. The boundary conditions are a geotherm derived by *Hofmeister* [1999] for oceanic lithosphere on the edges of the slab that are shallower than 80 km which corresponds to the depth where the geotherm intersects the peridotite solidus. The geotherm is approximately given by $T = 277 + 12.297z + 0.05733413z^2$ where T is temperature in kelvins and z is depth in kilometers. This is a simple parameterization of the geotherm based upon the numerical solution of the conduction equation using variable conductivity [*Hofmeister*, 1999]. Below 80 km depth a mantle adiabatic gradient of 0.3 K km^{-1} is used to constrain the boundary conditions on the wedge and forearc sides of the slab in the mantle. The down dip edge of the slab is modeled as a zero net heat flux boundary condition, which is appropriate for a steady-state model of this type because it is the same as imposing a temperature at infinite downdip length [e.g. *McKenzie*, 1969].

Thermal conductivity is variable following the formulation of *Hofmeister* [1999] and is given by:

$$k(P,T) = k_{1\,\mathrm{atm},298}\left(\frac{298}{T}\right)^{a} \exp\left[-\left(4\gamma + \tfrac{1}{3}\right)\int_{298}^{T}\alpha(\theta)d\theta\right] \times\left(1+\frac{K_0'P}{K_T}\right)+k_{\mathrm{rad}}(T) \qquad (2)$$

where $k_{1\mathrm{atm},298}$ (5.2, 6.5, and 8.8 W/(m K) for α, β, and γ phases respectively) is the thermal conductivity at standard conditions, γ is the thermal Grüneisen parameter, α is the volume coefficient of thermal expansion as a function of temperature, K_0 is the reference isothermal bulk modulus, $K'_0 = \mathrm{d}K_T/\mathrm{d}P$ where K_T is the isothermal bulk modulus, and k_{rad} is an estimate of the radiative contribution to the thermal conductivity. The exponent, a, is a fitting parameter from data for the temperature dependence of individual minerals. Equation (2) is valid under the following assumptions: (a) K'_0 = constant; (b) the variation of the bulk modulus as a function of temperature and pressure are mutually independent; and (c) $\mathrm{d}\gamma/\mathrm{d}P \approx 0$. For the parameter, a, we use 0.45, 0.4, and 0.33 for α, β, and γ respectively [*Hofmeister*, 1999; Hofmeister, personal communication, 1999]. For the sake of simplicity and to isolate the variable conductivity and the first-order effects of the jumps in conductivity at ~410 km and ~520 km, we model the transitions as α (olivine)→ β (wadsleyite)→ γ (ringwoodite). This simplified mineralogical structure is sufficient to describe the conductivity structure of the upper mantle and transition zone to first-order.

3. Results

To investigate the potential effects of variable thermal conductivity on a subducting slab, we performed numerical experiments that covered a range of values for slab velocity and slab dip. The age of the subducting lithosphere was not varied in this exploratory study, but was assumed to have reached conductive steady state (old lithosphere > 70 Ma).

Interdependence of temperature and thermal conductivity requires that we simultaneously solve for both variables. Figure 1 is a typical example our results. The variable conductivity model (Figure 1b-c) is warmer at the same depth compared to the constant conductivity solution. The effect of the mineralogy on conductivity at the β and γ phase transitions is striking (Figure 1c). The increase in thermal conductivity down dip across the phase transitions is greater than 50%, and is due to the higher conductivity of higher pressure phases (i.e. spinel) relative to olivine [e.g. *Fujisawa et al.*, 1968; *Hofmeister*, personal communication, 1999]. Higher thermal conductivity suggested for the transition zone (defined here as the region where β and γ phases are present) below the 410 implies that in steady state the slab should be hotter than expected under typical constant conductivity models [e.g., *Devaux et al.*, 1997]. Dimensionally equation (1) says that the length scale for temperature penetration of the slab is $L = k_{eff}/\rho C_P V$, where k_{eff} is the effective thermal conductivity. Hence, below the 410, the mantle temperature has penetrated more deeply into the slab.

McKenzie [1969] showed that the depth of a particular isotherm in a subducting slab is proportional to the slab velocity. Figure 2 compares the results from our numerical experiments for the thermal structure of slabs with variable conductivity to those with a constant conductivity as a function of slab velocity and dip angle. The dashed lines represent the 1:1 correlation lines for constant versus variable conductivity models. Figure 2a compares the depth to the 650°C isotherm (as a simple proxy for temperature structure) and Figure 2b shows the length of a putative metastable olivine wedge assuming for simplicity that the 650°C isotherm controls the metastable $\alpha \rightarrow \beta$ transition [i.e. *Sung and Burns*, 1976; *Daessler and Yuen*, 1996; *Kirby et al.*, 1996]. The wedge length in Figure 2b is the difference between the depth of the 650°C isotherm and the depth of the uplifted equilibrium $\alpha \rightarrow \beta$ phase boundary. Approximation of the metastable $\alpha \rightarrow \beta$ transition by a critical isotherm appears to be justified by previous work [i.e. *Sung and Burns*, 1976; *Daessler and Yuen*, 1996; *Kirby et al.*, 1996].

Both depth to the 650°C isotherm (Figure 2a) and wedge length (Figure 2b) show that the predicted thermal structure of subducting slabs with variable conductivity deviates from that predicted for constant thermal conductivity. The most significant deviation appears at high slab velocities and with increasing dip. Models that are to the left of the 1:1 correlation lines (Figure 2) indicate that variable conductivity models predict hotter deep slabs than constant conductivity models (mineralogy plays the dominant role). Conversely, models that are situated to the right of the correlation lines suggest cooler slabs than constant conductivity models would predict (pressure- and temperature-dependence of conductivity play the dominant role). In general, for slab velocities less than ~8 cm/yr it appears that variable conductivity predicts slightly cooler slabs than a constant

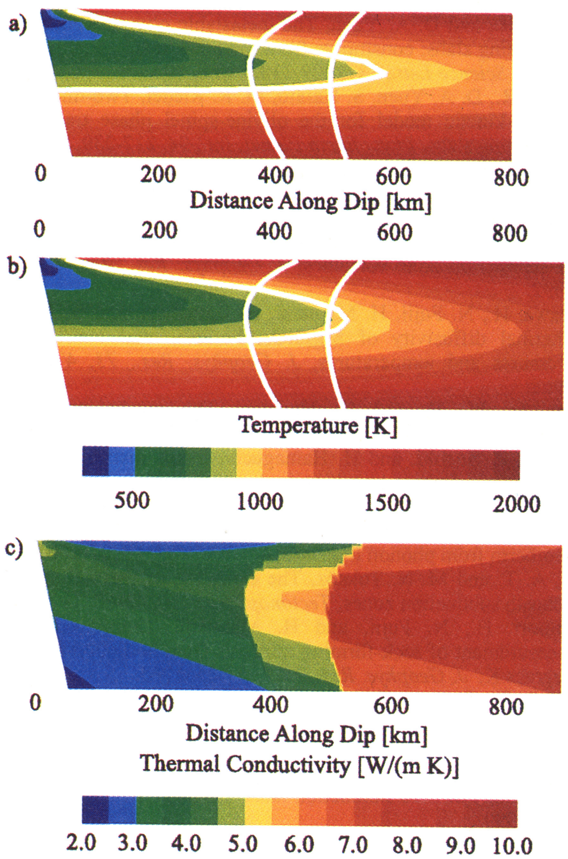

Figure 1. Contour plots of solutions for temperature, a) and b), and thermal conductivity, c), for an 80 km thick slab (vertical dimension) subducting at 8 cm/yr at a 70° dip. Horizontal axis is distance along dip of the slab. The white lines curved to the left represent the phase boundaries, while the one curved to the right outlines the 650°C isotherm. a) Thermal solution for a constant k=3.5 W/(m K). b) Thermal solution for variable conductivity. c) Conductivity solution for corresponding to b). Note the shorter distance between the equilibrium $\alpha \rightarrow \beta$ boundary and the 650°C isotherm for a) compared to b).

conductivity of 3.5 W/(m K). At higher velocities and with increasing slab dip, the opposite appears to be true. For a slab with a velocity of 12 cm/yr and dipping at 70° the deviation is of the order of ~25% for the depth to the 650°C isotherm and ~40% for the putative 'wedge' length. Calculations were also performed with a univariant transition with an average jump in $k_{1atm,298}$ (5.2 and 7.7 W/(m K) for α, $\beta + \gamma$ phases respectively) as well as with values for the $k_{1atm,298}$ for β and γ of 5.7 and 9.2 W/(m K) which may agree more closely with experimental results [Hofmeister, personal communication, 1999]. These additional calculations provided solutions indistinguishable from those presented in Figure 2, which demonstrates the robustness of the result.

4. Discussion and Conclusions

The results presented above suggest a revised view of the details of the thermal structure of subducting slabs, particularly ones subducting rapidly at high dip angles. Use of *Hofmeister's* [1999] conductivity formulation, which seems to agree well with a range of materials from salts to silicates to oxides, introduces some interesting variation into the heat transfer process within slabs. Figure 1 provides a convincing view of this process through the temperature and conductivity structure. Lateral variations (perpendicular to the slab dip direction) of conductivity (due to the pressure- and temperature-dependence of conductivity) suggest that the process of slab warming is more complicated than previously thought. As the outer layers of the slab warm (at the top or upper surface of the slab, the bottom of the slab already being warm) at shallow levels in the mantle, the thermal

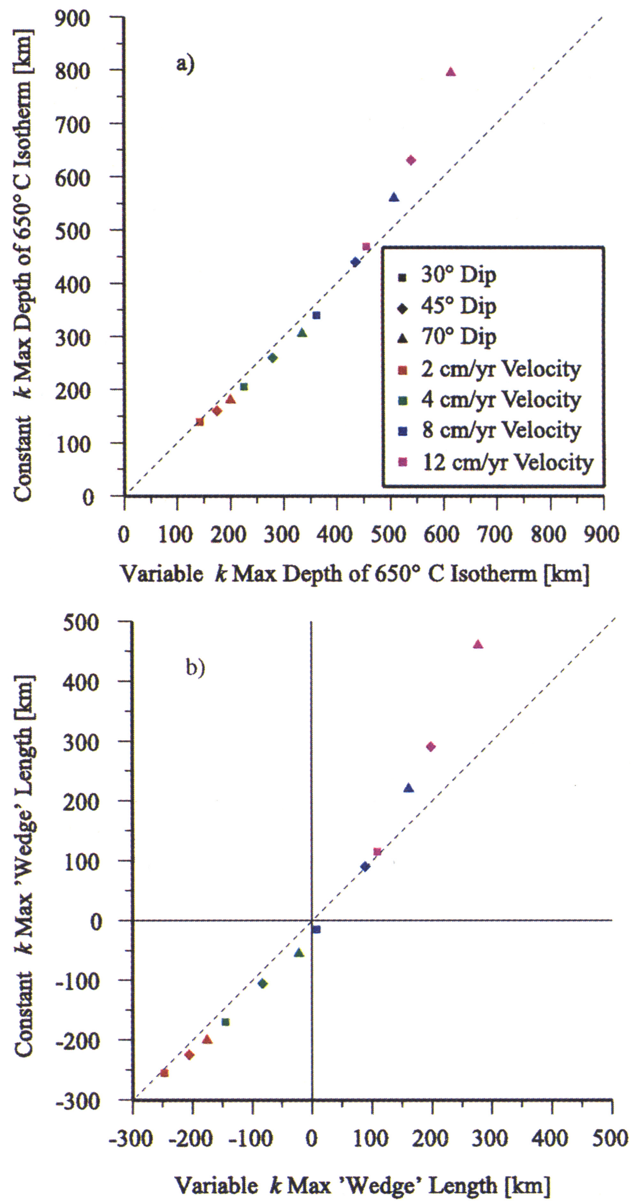

Figure 2. Comparisons of steady state constant thermal conductivity models versus variable conductivity models: a) depth to the 650°C isotherm, b) distance between the uplifted 410 km equilibrium phase boundary and the 650 °C isotherm. The symbol shape shows dip angle and the velocity is depicted by color. The dashed line is the 1:1 correlation line between constant [k = 3.5 W/(m K)] and variable conductivity models.

conductivity will decrease. This decrease in conductivity will impede the diffusion of heat into the core of the slab.

The phase transition that begins at ~410 km depth introduces discontinuities in many material properties, including the thermal conductivity. The increases in thermal conductivity across the $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$ phase boundaries result in a hotter slab in the transition zone because conduction is more efficient in the γ stability field than the α and β fields. Jumps in conductivity in the transition zone appear to be significantly more important for temperature structure than the lateral variation in conductivity, especially for fast slabs, as is evident directly from Figure 2. In addition, Figure 2 shows that assuming constant conductivity significantly underestimates the temperature of fast, steeply dipping slabs. The effect of the lateral variation is small at slow slab velocities, whereas the effect of the discontinuity is quite significant at fast slab velocities.

Future work should include the effects of a metastable olivine wedge on the thermal structure of subducting slabs. A metastable tongue of olivine below the 410 km equilibrium discontinuity will result in a locally lower thermal conductivity. In order to do this problem well, the kinetics of the $\alpha \rightarrow \beta$ phase transformation should be included, but the effect of variable conductivity is smaller than the current uncertainty in the kinetic reaction constants [e.g. *Daessler and Yuen*, 1993; *Devaux et al.*, 1997]. With better kinetics data, the complex interplay between the kinetics and variable conductivity would provide interesting information on the thermal structure of slabs in the transition zone.

The realization and introduction of a more complex, yet realistic, formulation for the variation of thermal conductivity within the Earth [*Hofmeister*, 1999] has some interesting implications for subducting slabs. For fast, old slabs that are dipping steeply, it is readily apparent that constant thermal conductivity models underestimate the temperature. A significant implication is that a model that reaches thermal steady state and uses a constant conductivity assumption would overestimate the depth extent of a potential metastable olivine wedge. Whereas the general understanding of the thermal structure of subducting slabs remains unchanged, it is obvious that use of conductivity that depends upon temperature, pressure, and mineralogy can significantly alter the details of the thermal structure of deep subducting slabs.

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