Polarization Properties of Optical Components

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Abstract

Millimeter wave optical components are useful for current Cosmic Microwave Background Radiation (CMB) experiments in the field of Cosmology. Therefore, it is useful to characterize these components. The purpose of this study is to create a millimeter wave system capable of testing the polarization properties of such components. This millimeter wave system will be constructed out of a Gunn oscillator opposite a diode detector with the device to be tested in the middle. In order to make sure that only photons of a certain polarization reach the millimeter wave optical components, a polarization grid will be placed both before and after this device. These polarization grids will be designed, built, and tested for the purpose of this study. The grids will be used to control the polarization state of both the incident wave on the device under test, and the transmitted wave that makes its way to the detector. Once this system is constructed, it will be used to test a half-wave plate around the 95 GHz frequency range.

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1. Introduction and Motivation

Cosmic Microwave Background Radiation, or CMB, is the afterglow of the Big Bang that fills the skies with a light of nearly uniform intensity. Studying the CMB is useful in learning about the early universe and even predicting how the universe is expanding. The CMB is also polarized. Studying this polarization signal helps us learn about what kinds of waves and other objects from the early universe caused them.

Telescopes which study the polarization signal from the CMB are equipped with polarization detectors. These polarization detectors gauge the linear polarization of incoming light. Linear polarization of a propagating wave is defined by the direction of its electric field. It’s also perpendicular to the direction of the wave in the same direction as its propagation. Since a wave which is propagating perpendicular to a particular polarization has no component of polarization in that direction, polarization detectors must be able to measure two perpendicular components of polarization. Polarization detectors need a number of components that deal with linear polarization since this is the only type of polarization expected from the CMB. One component commonly used for polarization detectors is called a half-wave plate.

Three different types of polarization

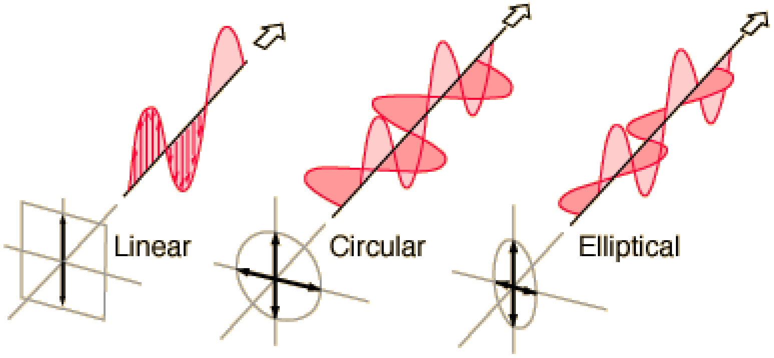
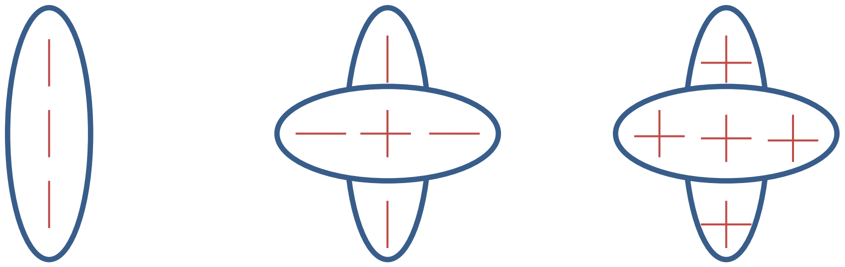


Figure 1

2. Half-wave Plates

Half-wave plates are optical devices that play an integral part in polarization detectors. Half-wave plates are birefringent mediums that rotate the polarization of incoming light. If the incoming light is θ degrees off the fast axis of the half-wave plate, it is transmitted at an angle of 2θ from the incoming light. There is one major reason that this is important for polarization detectors. Typically polarization detectors can only detect one polarization of light in a non-ideal elliptical pattern. Polarization detectors can be stacked on top of each other such that there is a detector measuring a vertically polarized signal in a vertically elliptical pattern and a detector measuring a horizontally polarized signal in a horizontally elliptical pattern overlapping the vertically elliptical pattern. This means that we will have an area in the middle that is detecting both the vertical and horizontal component of polarization but four areas that are only detecting one polarization component. Unfortunately, in these four areas, there is a possibility that other signals in the CMB could be mistaken for polarization signals of the one component. Half-wave plates correct for this by rotating the linearly polarized light 90 degrees so that the vertical detector is now measuring the horizontal polarization in the vertical elliptical pattern and the horizontal detector is measuring the vertical polarization in the horizontal elliptical pattern.



A) a detection pattern of one polarization

B) overlapping detection patterns of one polarization

C) detection pattern for overlapping detectors with a half-wave plate

Figure 2

3. Methods

It was my intent for this project to create a millimeter wave system that can test optical components for use in polarization detectors. In particular, the system would be capable of testing polarization properties as well as attenuation properties. I intended to only test one half-wave plate, but I intended for it to be created so that it can easily test other optical components for use in polarization detectors.

3.1 Preparation

Constructing a millimeter wave system to test polarization and attenuation properties requires a Gunn oscillator to emit photons at a specific frequency. These photons pass through a lens which focuses them through a polarization grid. The polarization grid filters out undesired polarization by only allowing one specific polarization of light through. This light then goes the half-wave plate, which rotates the polarization of the light. The light travels again through a polarization grid with the same orientation as the first, grid and is finally focused through a lens into the Gunn diode. The Gunn diode measures the intensity of the light in combination with a lock-in amplifier. The lock-in amplifier locks into the frequency of the Gunn oscillator to ensure that the signal is only coming from the desired photons. This same setup could be used in the future with other components that need to be tested in place of the half-wave plate.

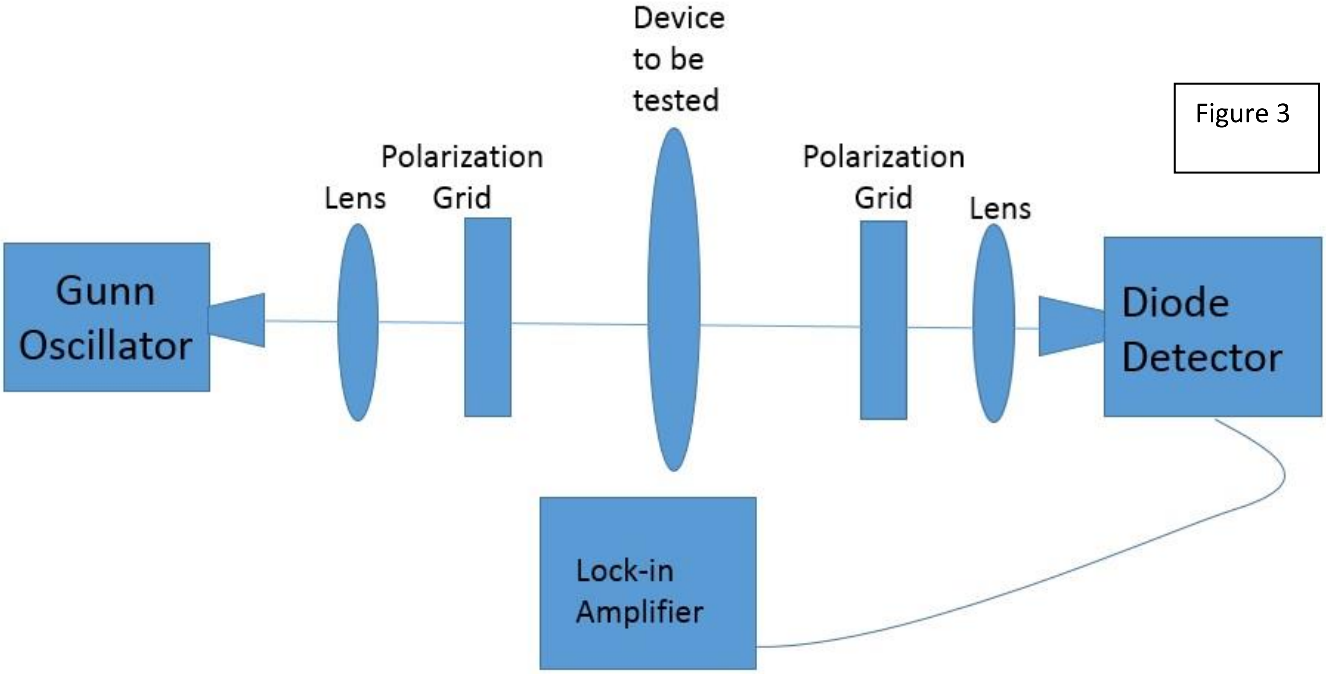
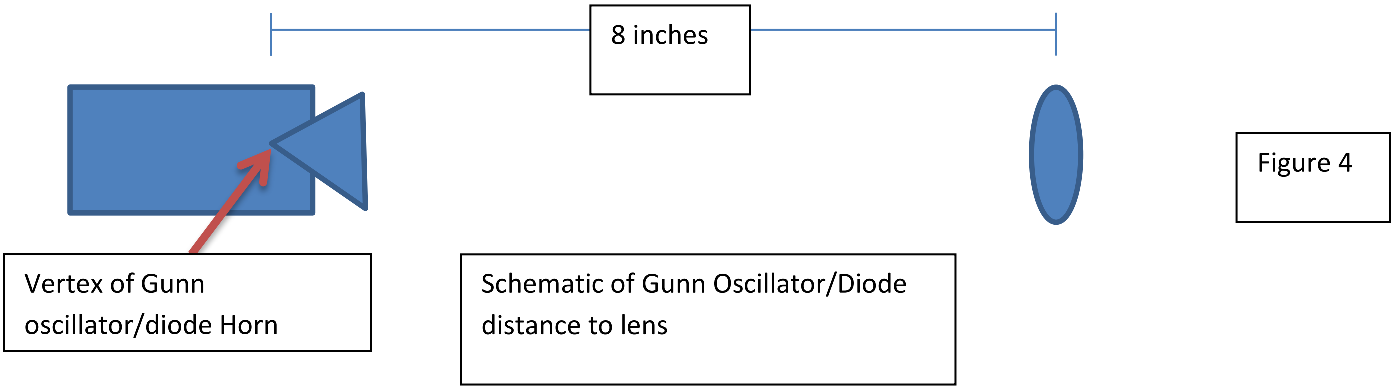


Figure 3

Schematic of the millimeter wave system

It is desired for this system to be integrated together as much as possible to ensure little room for misalignment. To that end, it was useful to model all of the pieces of the assembly on Solidworks in order to plan out and construct an assembly that could integrate together and support the necessary components. In constructing the model, it is important to make both the Gunn oscillator horn vertex and the Gunn diode horn vertex the correct distance from their respective lenses. This distance is determined by the focal length of the lens. For this project, the assembly is constructed with the Gunn oscillator/Gunn diode horn vertexes eight inches from the center of their respective bi-convex lenses.



Schematic of Gunn Oscillator/Diode distance to lens

Figure 4

The waveguide of the Gunn oscillator, which is the piece which the photons travel through before being emitted through the horn, is welded to a small circular piece that attaches to the cylinder assembly such that the Gunn oscillator horn is inside of the cylinder. On the other side of the cylinder, the lens and polarization grid are attached separately so that the polarization grid can be taken off without removing the lens. The Gunn diode side of the assembly is identical except that the body of the Gunn diode can hang outside of the cylinder assembly as it is light enough to not require support.

Solidworks Model of the Gunn Oscillator side of the assembly

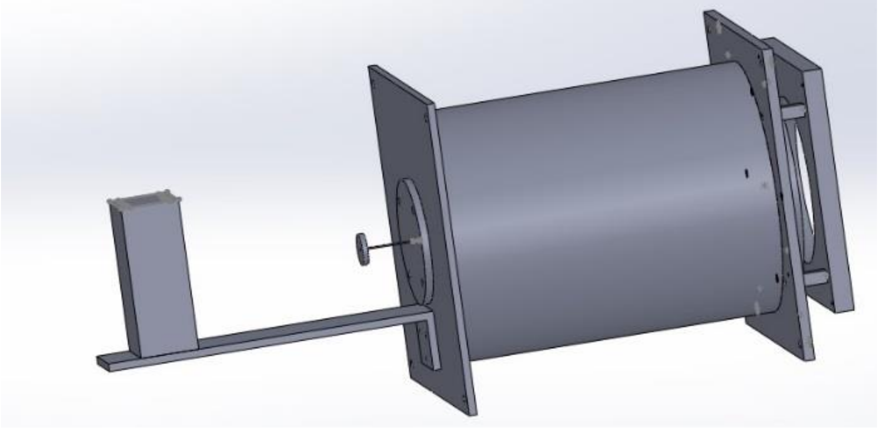


Figure 5

The whole assembly was created out of aluminum so that it was light enough to support easily. The insides of the cylinders were covered with Eccosorb HR-10. This lightweight foam absorbs the millimeter waves so that they do not reflect off of the metal insides of the cylinder. At this point, everything for the assemblies was ready except for the polarization grids, as they did not yet exist.

3.2 Creating Wire Grids

Polarization grids are not mass produced and certainly unlikely to be readily available with a specific sized frame fitting a specific system. It turned out to be easier to create polarization grids. The polarization grids will be designed, created, and tested for this project. The idea of these polarization grids is to only allow emitted photons of a certain polarization to be transmitted through them. In this way, the polarization of light that reaches the half-wave plate and the Gunn diode can be controlled and perhaps more importantly, is known.

For this project, the polarization grid consists of a square solid frame with a four-inch diameter hole. This hole is spanned by one mille-inch diameter gold-plated tungsten wire which is wound across the whole frame. The wire is wound around the frame at specific constant spacing. This spacing is designed to allow only photons propagating perpendicular to the wires to transverse through the grid. Light waves carry an electric field with them in a direction defined as their polarization. The electric field of the parallel propagating photons induces the electrons to move along the wires, causing a vibration. This vibration occurs at a frequency greater than the frequency of the photons, effectively creating a sheet of metal which reflects the photons. On the other hand, for perpendicularly propagating photons, the electric field causes the electrons to move perpendicular to the wires which doesn't cause a vibration due to the thin properties of the wire. Thus, the photons pass right through the grid.

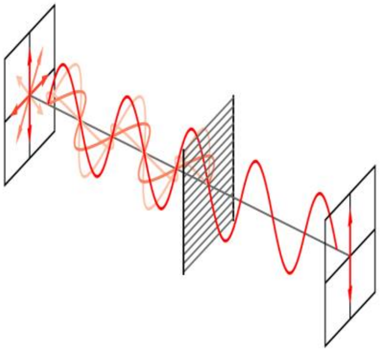


Figure 6

An illustration of a polarization grid at work

As can be found in the textbook by Lesurf [7], the reflection coefficient of the parallel and perpendicular (or normal) polarizations are dependent on three parameters: the wavelength of the photons (λ), the spacing between the wires (S) and the wire diameter (d).

$$\left|r_p\right|^2 = \frac{1}{1 + \left(\frac{2S}{\lambda}\right)^2 ln\left(\frac{S}{\pi d}\right)^2}$$

Equation 1

$$|r_n|^2 = \frac{(\pi^2 d^2)^2}{\lfloor 2\lambda S \rfloor^2 \left[1 + \frac{(\pi^2 d^2)^2}{(2\lambda S)^2}\right]}$$

Equation 2

For this project, the wire used is .001 inch gold-plated tungsten. The goal is to find a spacing (S) that would be optimal for the range of frequencies in the lab. These equations can be plotted at the wavelength corresponding to 83.3 GHz as a function of the spacing (S) in Matlab. By looking at these plots, the optimal spacing can be determined by finding a point that optimizes the parallel reflection coefficient (r_p) and minimizes the perpendicular reflection coefficient (r_n).

Parallel reflection coefficient as a function of wire spacing

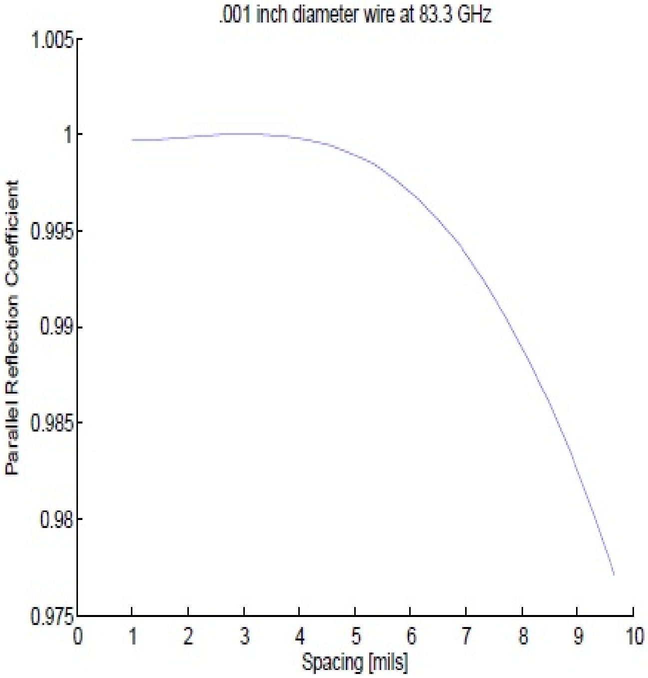


Figure 7

Perpendicular reflection coefficient as a function of wire spacing

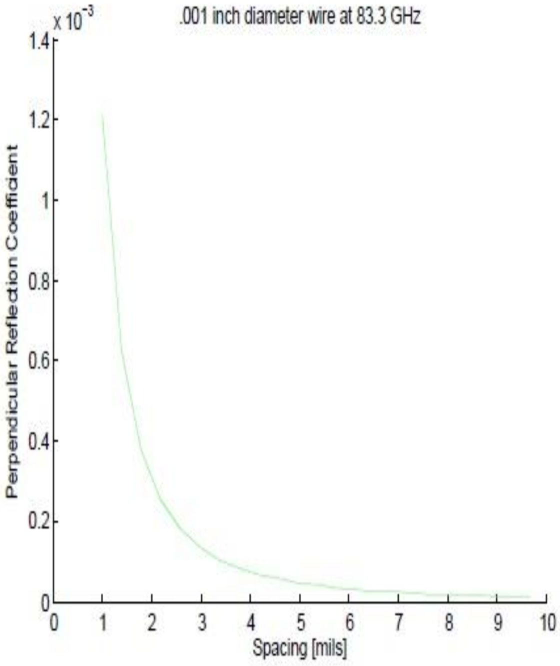


Figure 8

I found this point to be at about 3.94 milli-inches using a Matlab figure cursor. It is useful to see how the reflection coefficients r_p and r_n behave as a function of the frequency of the incoming photons. Plotting these functions determines whether the spacing chosen will work for a range of frequencies that will be used in the lab.

Parallel and perpendicular reflection coefficients as a function of frequency

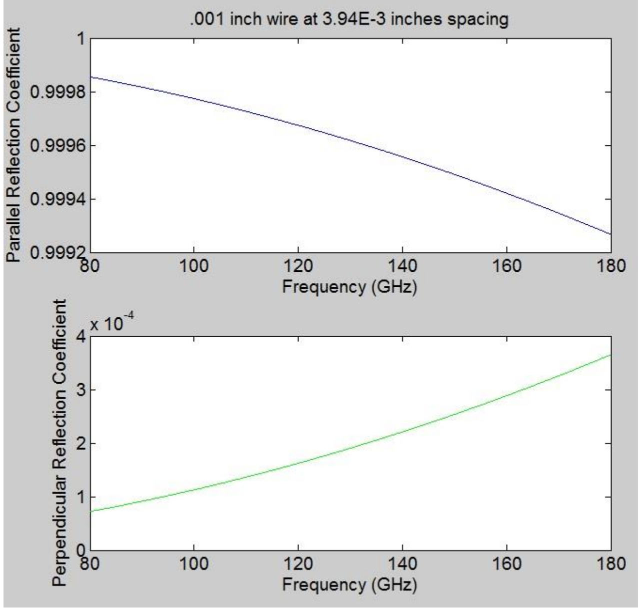


Figure 9

A look at the scale for the perpendicular and reflection coefficients reveals that the frequency has little effect on the reflection coefficients for this particular spacing. Now that the spacing is determined, the grids can be wound.

For this project, two frames were bolted together so that winding once would actually create two polarization grids. It works by slowly rotating the frames so that the wire is wrapped around the frames. The wire is fed from a coil in through a pulley and finally through a component that gradually moves alongside the grids at a speed that gives the proper center-to-center separation between the wires. The sides of the frames were taped with double sided tape so that the wire would wrap around the assembly and stick to it. There is a motor connected to the wire coil which offers resistance to the rotation of the polarization. The motor, which was purchased after seeing a similar setup for another grid winder [2], varied its speed according to the shape of polarization grids to give a nearly uniform tension in the wire at all times. Using this setup, the grids can be wound very slowly so as to not break the wire.

A picture of the completed polarization grid

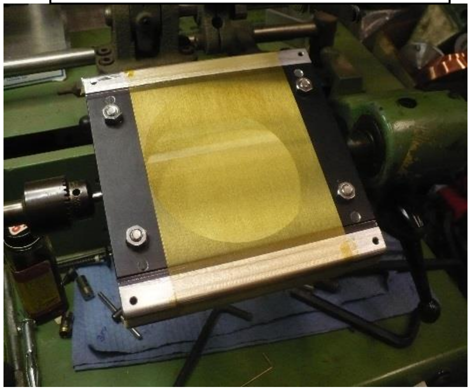


Figure 10

Once created, the polarization grids have to be tested to ensure that they have the desired effect. For this project, the polarization grids were tested by putting them in the millimeter wave system shown in figure ? oriented with their wires running horizontally and taking out the device to be tested. The intensity registered by the lock-in was recorded for 90, 95 and 100 GHz. The setup was the adjusted by rotating the second polarization grid from the left 90 degrees.

Theoretically, the photons coming out of the first polarizer would be propagating vertically and when they arrive at the second polarizer, which is now oriented with the wires running vertically, they would be completely reflected. The intensity registered by the lock-in was once again recorded for 90, 95, and 100 GHz. At each frequency the intensity registered in the second setup was less than 1% of the intensity registered in the first setup. This is a strong indication that the polarization grids are working properly and the half-wave plate can be tested with the millimeter wave setup.

3.3 Taking Data

With the setup completed, the half-wave plate is inserted into a frame with metal teeth that is driven by a stepper motor. This frame rotates the half-wave plate around in a precise manner according to the stepper motor. The half-wave plate is adjusted until there is a maximum of the intensity measured by the lock-in amplifier. This indicates that the polarization of the light is aligned with the fast axis of the half-wave plate. A Matlab program powers the stepper motor through a serial port. The Matlab program causes the stepper motor to rotate the half-wave plate 11.25 degrees and then pause for 10 seconds for two full rotations. All the while, the Matlab reads in data from the lock-in to a text file for future analysis. This experiment was run once at 90 GHz, once at 95 GHz, and once at 100 GHz. The three text files are then read into a different Matlab file for analysis.

A picture of the completed millimeter wave system with the half-wave plate in place

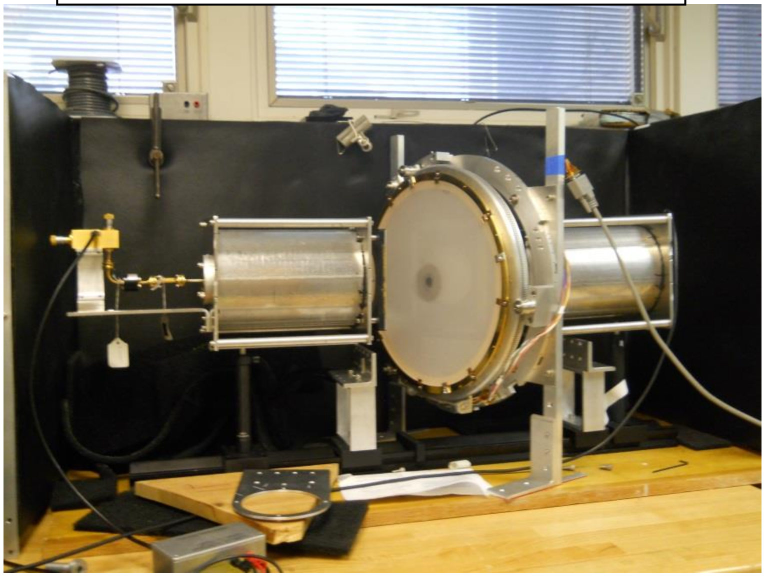


Figure 11

4. Results and Analysis

The Matlab program reads in the text files for experimental runs at the three distinct frequencies. It plots the intensity from the lock-in as a function of the number of data points that have been recorded. Since the half-wave plate takes about ten seconds to rotate, there are data points during these transition times that would best be eliminated since the lock-in cannot get a stable read-out during rotation. Those data points can be eliminated by finding them in the Matlab plot and then re-graphing intensity from the lock-in as a function of the discrete angle. These data points can then be compared to the theoretical curve of the “ideal” half-wave plate by using Jones calculus to find this theoretical curve.

Jones calculus dictates that the polarization of light (P_i) can be represented as a 2x1 vector called a Jones vector:

$$P_i = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

Equation 3

E_x and E_y refer to the x and y components respectively of the electric field. The coordinates are set with the light traveling along the z-axis, the horizontal along the x-axis and the vertical along the y-axis. For this project, the light is then acted on by a linear vertical polarization grid (G_v), which can be represented by a Jones matrix:

$$G_v = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Equation 4

After first polarization grid, the light is acted on by the half-wave plate (H) which, according to theory, rotates the lights polarization by two times the angle (θ) between the fast axis of the half-wave plate and the polarization of light. This rotation can be represented by another Jones matrix:

$$H = \begin{bmatrix} -\cos(2\theta) & -\sin(2\theta) \\ -\sin(2\theta) & \cos(2\theta) \end{bmatrix}$$

Equation 5

And according to the millimeter wave system, the light hits one more vertical polarization grid before it is detected by the Gunn diode. Putting all this together we can get a vector expression of the final polarization of light:

$$P_f = G_v(H(G_v P_i)) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} -\cos(2\theta) & -\sin(2\theta) \\ -\sin(2\theta) & \cos(2\theta) \end{bmatrix} \left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} \right) \right) = \begin{bmatrix} 0 \\ E_y \cos(2\theta) \end{bmatrix}$$

Equation 6

For this project, however, the intensity of the light was measured rather than the electric field. The above equation can be altered to find the final intensity (I_f) as a function of θ, by taking the square root of the initial intensity (I_i) and squaring the final result:

$$I_f(\theta) = \left[\sqrt{E_y}\cos(2\theta)\right]^2$$

Equation 7

Finally, plotting the theoretical result for an ideal half-wave plate over the data gathered with the lock-in amplifier gives the following figures:

Figure 12: Intensity measured from lock-in as a function of angle of HWP at 90 GHz

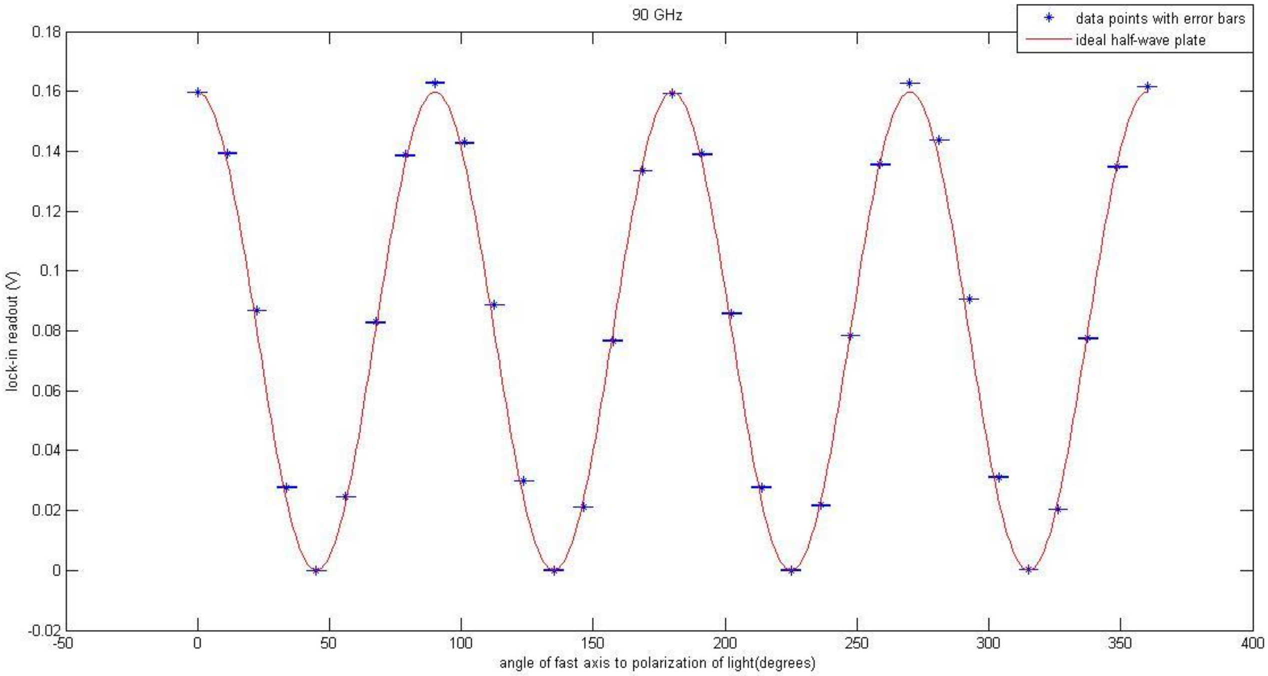


Figure 13: Intensity measured from lock-in as a function of angle of HWP at 95 GHz

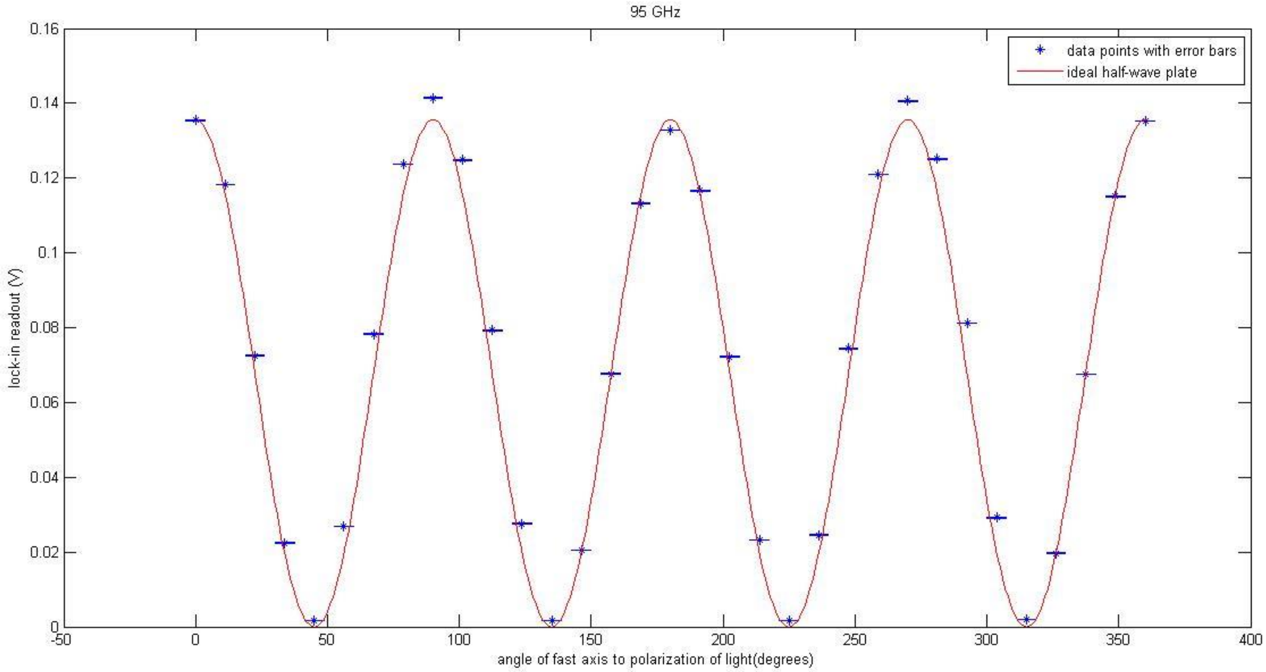
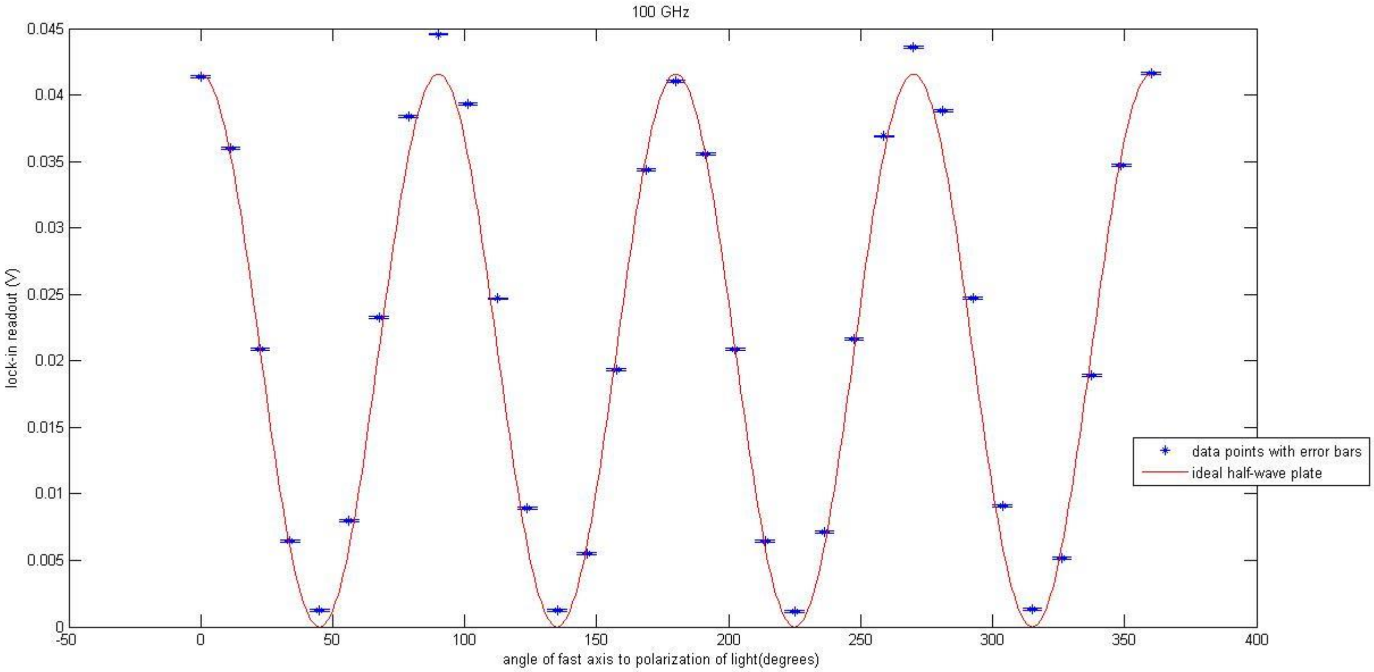


Figure 14: Intensity measured from lock-in as a function of angle of HWP at 100 GHz



As can be seen from the plots, the blue stars are the discrete data points at every 11.25 degrees and the red line is the theoretical result for an ideal half-wave plate. Both are plotted as a function of the angle. The error bars from the data points are only slits because they are very small due to the consistency with which the lock-in measured the intensity.

From looking at these plots it is easy to see that the half-wave plate reacts nearly as expected for an ideal half-wave plate. Unfortunately, in reality, there are typically several non-idealities involved in half-wave plates.

The half-wave plate used for this project is made of a single crystal sapphire plate at a thickness that is approximately half of the wavelength of the photons for which the half-wave plate is built. In this case, the half-wave plate is built for 95.6 GHz. The sapphire is a birefringent medium with indices of refraction of 3.336 and 3.019. The birefringent nature of the half-wave plate causes incoming light of differing polarizations to be rotated to varying degrees. This rotation turns out to be twice the angle of the polarization of the incoming light to the fast axis of the half-wave plate. The high indices of refraction of the half-wave plate mean that there is risk of a lot of reflection from both sides. To account for this, an anti-reflective coating is applied to both sides of the half-wave plate. The coating is approximately as thick as a quarter of the wavelength of the photons for which the half-wave plate is built. This specific thickness reduces reflections because the relative path length difference between the reflections from the front and back surface will be a half-wavelength (due to the thickness of the crystal) and so will destructively interfere. For the reflections to completely destructively interfere, however, they must have the same amplitude. It turns out that this occurs when the index of refraction for the anti-reflective coating is $\sqrt{n}$ where n is the index of refraction of the half-wave plate (in this case we get a range of values since there are two indices of refraction). Unfortunately, when this half-wave plate was made, an anti-reflective coating with an index of refraction in that range was not available. Sean Bryan [4], estimates that this half-wave plate with its imperfect anti-reflective coating will still have reflections of about 3.2%. Another problem with these anti-reflective coatings is that they require an adhesive to stay on the half-wave plate which changes the way the half-wave plate works.

It is also worth mentioning that it appears the lenses of the millimeter wave system are out of focus. While the intensity was being measured by the lock-in, the half-wave plate was taken in and out of the system with the fast axis aligned with the polarization. In theory, this should have no effect as the lenses have already focused the photons such that the diode is receiving the maximum intensity. However, in reality, the intensity measured by the lock-in was slightly higher with the half-wave plate in. The intensity measured with the half-wave plate in was about .01 volts higher. This is probably because the lenses were not at quite the right distance from the Gunn oscillator and Gunn diode respectively. As a result, the indices of refraction of the half-wave plate may have actually helped focus the photons out of the Gunn diode and into the Gunn oscillator. There are a couple of reasons the lenses could be out of focus. The precision required to make cylinders the correct lengths may have been too hard to be possible in reality. There also may have been mistakes with measurements of distances that were important to the focus of the lenses. However, the distance between the horns and the lenses was measured when the system was first created and appeared to be correct. This suggests it’s also possible that the lenses focal length was slightly different from the focal length that was advertised.

5. Conclusions

The collected data was very encouraging. It revealed that the half-wave plate works nearly to perfection, at least to how it is supposed to work. Of course the data is not perfect, but this is a result of some of the non-idealities of the half-wave plate. It’s important to note that the discrepancies between the data collected at 90, 95 and 100 GHz were a little bit different. This is actually reassuring as the half-wave plate is made to work best for a particular frequency which is determined by the thickness of the sapphire. In this case, the half-wave plate was made for about 95 GHz.

Perhaps more importantly, the collected data shows that the polarization grids which were created for this project worked well. If the grids were flawed, the data that was collected would not have correlated nearly as well to the theoretical curves. Polarization grids that work for a specific system are very valuable as it is expensive in terms of both time and money to have them custom-made. The data gathered not only helps characterize the half-wave plate, but confirms the use of the millimeter wave system. This system could turn out to be very useful for future experimentation.

6. Future Work

As I mentioned in the Analysis section, there appears to be a problem with the focusing of the lenses in the millimeter wave system. The problem should be easily mixed by testing the correct distance required and adding spacers to push the lenses farther from the horns to optimize the lock-in read out. If the lenses need to actually be closer to the horns, the waveguide attached to the horn would need to be elongated.

It is my hope that this system's use did not end with the data I collected. The millimeter wave system can test both polarization and attenuation properties of optical components and so could be used for something other than just a half-wave plate. I hope it has a continued use testing optical components which are sensitive to polarization.

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