Mechanics Formulae for Physics Proficiency Exams

Motion, forces, work, energy and momentum.

$v = dx/dt$ $a = dv/dt$ $v = v_o + at$ $x = x_o + v_o t + at^2/2$ $v^2 = v_o^2 + 2ad$

$F = ma$ $f = \mu N$ $W = \mathbf{F} \cdot \mathbf{x}$ $K = ½mv$

$U_g = mgh$ $U_{spring} = ½kx^2$ $F_x = -dU/dx$ $x_{cm} = \Sigma m_i x_i / \Sigma m_i$

$\mathbf{p} = m\mathbf{v}$ $\mathbf{F} = d\mathbf{p}/dt$ J or $I = \Delta p = \int F dt$

Rotational motion.

$\theta = s/r$ $\omega = d\theta/dt$ $\alpha = d\omega/dt$ $a_c = v^2/r = \omega^2 r$ $K = ½ I\omega^2$ $I = \Sigma m_i r_i^2$

$I_{disc} = ½MR^2$ $I_{sphere} = (2/5)MR^2$ $I_{rod\text{-}CM} = (1/12)ML^2$ $I_{parallel} = I_{CM} + MR^2$

$\tau = I\alpha$ $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$ $\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$ $L = I\omega$ $\Sigma\mathbf{F} = 0$ $\Sigma\boldsymbol{\tau} = 0$

Oscillators, gravity.

$F = -kx$ $E = ½kx^2$ $y = A\cos(\omega t + \phi)$ $\omega = 2\pi/T$ $T = 1/f$ $\omega = (k/m)^{1/2}$ $T = 2\pi(\ell/g)^{1/2}$

$F = GM_1M_2/r^2$ $U_g = -GMm/r$ $K = GMm/(2r)$

$T^2 = r^3(4\pi^2)/(GM)$

Fluids, waves, sound.

$\rho = m/v$ $p = F/A$ $p = \rho gh$ $p + 0.5\rho v^2 + \rho gh = const$ $\rho_{H2O} = 1\ g/cm^3$

$y = A\sin(kx - \omega t)$ $k = 2\pi/\lambda$ $\omega = 2\pi/T$ $v = \lambda f = \omega/k$ $v = (\tau/\mu)^{½}$

$\beta = 10\log(I/I_O)\ dB$ $f' = f(v+v_D)/(v+v_S)$

Thermo, heat, kinetic theory.

$T_F = 1.8\,T_C + 32$ $\Delta L/L = \alpha\,\Delta T$ $Q = cm\Delta t$ $Q = Lm$ $\Delta E_{int} = Q_{in} - W_{done}$

$\Delta Q/\Delta t = kA\,\Delta T/L$ $KE_{av} = 1.5\,kT$ where $k = R/N_A = 1.38 \times 10^{-23}\ J/K$

Constants.

$N_A = 6.02 \times 10^{23}$ $R = 8.31\ J/mol\text{-}K$ $G = 6.67 \times 10^{-11}\ N\,m^2/kg^2$ $g = 9.8\ m/s^2 = 32\ ft/sec^2$

MISC.

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Electricity & Magnetism Formulae for Physics Proficiency Exams

Constants:

$e = 1.6 \times 10^{-19}$ C $k = 1/(4\pi\varepsilon_0) = 9 \times 10^9\ N\,m^2/C^2$

$\varepsilon_o = 8.85 \times 10^{-12}$ F/m $\mu_o = 4\pi \times 10^{-7}$ T m/A

$c = 3 \times 10^8$ m/s 1 eV = 1.6×10^{-19} J $m_e = 9.11 \times 10^{-31}$ kg

Electric fields, resistance, circuits, capacitors:

$F = kQ_1Q_2/r^2$ $\mathbf{E} = \mathbf{F}/q$

Gauss's Law: $\Phi_E = \int \mathbf{E} \cdot \mathbf{A} = Q_{encl}/\varepsilon_o$ $\Delta V = -\mathbf{E} \cdot d\mathbf{r}$ $V = kq/r$ $U = \Sigma k q_i q_j / r_{ij}$

$-\mathbf{E} = \mathbf{i}\, \partial V/\partial x + \mathbf{j}\, \partial V/\partial y + \mathbf{k}\, \partial V/\partial z$ $i = V/R$ $R = \rho L/A$ $\rho - \rho_O = \rho_O\, \alpha\, \Delta T$

$P = iV$ Kirchhoff: $\Sigma_{loop} V = 0$ $i_{in} = i_{out}$ $\vec{\tau} = \vec{p} \times \vec{E}$

$q = CV$ $C = \varepsilon_o A/d$ $U_c = ½\, CV^2$ $q = q_o\, e^{-t/RC}$ or $q = q_f(1 - e^{-t/RC})$ $\tau_c = RC$

$R_{series\ equiv} = R_1 + R_2 + \cdots$ $1/R_{parallel\ equiv} = 1/R_1 + 1/R_2 + \cdots$

$C_{parallel\ equiv} = C_1 + C_2 + \cdots$ $1/C_{series\ equiv} = 1/C_1 + 1/C_2 + \cdots$

Magnetic fields and electromagnetism:

$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$ $\mathbf{F}_B = i\mathbf{L} \times \mathbf{B}$ $qvB = mv^2/r$

Biot-Savart Law: $d\mathbf{B} = (\mu_0/4\pi)(i\, d\mathbf{s} \times \mathbf{r})/r^3$

$\varepsilon = \oint \vec{E} \bullet d\vec{l} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(\vec{B} \bullet \vec{A}\right)$ Ampere's Law: $\oint \vec{B} \bullet d\vec{l} = \mu_o \varepsilon_o \frac{d\Phi_E}{dt} + \mu_o i$

$B_{line} = \mu_0 i / (2\pi r)$ $B_{hoop} = \mu_0 i / (2r)$ $B_{solenoid} = \mu_0 n i$

$\mathcal{E} = -L\, di/dt$ $L_{solenoid} = \mu_o n^2 A$ $\tau_L = L/R$ $U_L = ½\, Li^2$ $\omega^2 = 1/(LC)$

Optics:

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ $1/p + 1/i = 1/f = 2/r$ $1/f = (n-1)\,[r_1^{-1} - r_2^{-1}]$ $\sin\theta = 1.22\, \lambda/d$

$\vec{E} = E_{max} \sin(kx \pm \omega t)\,\hat{y}$ $k = 2\pi/\lambda$ $\omega = 2\pi f$ $\omega = ck$ or $c = f\lambda$ $\lambda_{effective} = \lambda/n$

$c = (\varepsilon_o \mu_o)^{-1/2}$ $E_{max} = B_{max} c$ $\mathbf{S} = \mathbf{E} \times \mathbf{B}/\mu_o$ $\Delta p = \Delta I/c$

Math:

The solutions to a quadratic equation: $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Surface Area of a Sphere is: $4\pi r^2$ Volume is $4\pi r^3/3$

Law of Cosines: $C^2 = A^2 + B^2 - 2AB\cos\theta_C$

Modern Physics Formulae for Physics Proficiency Exams

Constants

$\mu_B = eh/(8\pi m) = 9.274 \times 10^{-24}$ J/T

$e^2/(4\pi\varepsilon_0) = 1.440$ eV nm 1 u (or amu) = 931.5 MeV/c^2 $e = 1.6 \times 10^{-19}$ C

$k = 1/(4\pi\varepsilon_O) = 9 \times 10^9$ N m^2/C^2 $\varepsilon_o = 8.85 \times 10^{-12}$ C^2/(N m^2)

$N_{Avogad} = 6.02 \times 10^{23}$ $\mu_o = 4\pi \times 10^{-7}$ T m/A

Rest mass of an electron = 9.109×10^{-31} kg = 0.511 MeV/c^2

$c = 2.99792458 \times 10^8$ m/sec $hc = 1240$ eV nm

$h = 6.626 \times 10^{-34}$ J-s = 4.1356×10^{-15} eV-s $\hbar = 1.05 \times 10^{-34}$ J-s = 6.58×10^{-16} eV-s

1 eV = 1.602×10^{-19} J $k_B = 1.38 \times 10^{-23}$ J/K = 8.62×10^{-5} eV/K

Special Relativity

$\beta = v/c$; $\gamma = (1 - \beta^2)^{-1/2}$ $T = T_o\gamma$ $L = L_o/\gamma$

Binomial Expansion: $\gamma = 1 + \beta^2/2 + 3\beta^4/8 + \ldots$

Lorentz: $x' = \gamma(x-\beta ct)$ $y' = y$ $z' = z$ $ct' = \gamma(ct - \beta x)$

or

$$x' = \gamma(x - ut) \qquad x = \gamma(x' + ut')$$
$$y' = y \qquad y = y'$$
$$z' = z \qquad z = z'$$
$$t' = \gamma\left(t - \left(\frac{u}{c^2}\right)x\right) \qquad t = \gamma\left(t' + \left(\frac{u}{c^2}\right)x'\right)$$

velocity transform: $v_x' = (v_x - u)/(1 - v_x u/c^2)$ or $u_x' = \dfrac{u_x - v}{1 - \dfrac{u_x v}{c^2}}$

Doppler: $\nu' = \nu\,[\,(1-\beta)/(1+\beta)\,]^{1/2}$

Kinematics: $E = \gamma mc^2$ $pc = \beta\gamma mc^2$ $E^2 = (pc)^2 + (mc^2)^2$ $K = E - mc^2$

$KE = (\gamma_u - 1)mc^2$ $p = \gamma_u mu$

Quantum Theory

Photoelectric effect: $K_{max} = h\nu - \phi$ de Broglie wavelength: $\lambda = h/p$

Blackbody radiation: $I = \sigma T^4$ $\lambda_{max} T = 2.898\ 10^{-3}$ m K

Compton scattering: $\lambda' - \lambda = (hc/m_e c^2)(1 - \cos\theta)$

Electron diffraction: $n\lambda = d \sin\phi$ Heisenberg: $\Delta p_x \Delta x \geq \hbar/2$ $\Delta E \Delta t \geq \hbar/2$

Schrödinger Equation: $-(h^2/8\pi^2 m)d^2\psi/dx^2 + U\psi = E\psi$ $P(x)dx = |\psi(x)|^2\,dx$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x) \qquad T_{wide\,barrier} \cong 16\frac{E}{U_o}\left(1 - \frac{E}{U_o}\right)e^{-2\left(\frac{\sqrt{2m(U_o - E)}}{\hbar}\right)L}$$

Infinite 1D square well: $E_n = (n^2\pi^2\hbar^2)/(2mL^2)$ $\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$ for $0 < x < L$

Infinite 3D square well: $E_n = (\pi^2 \hbar^2)(n_x^2 + n_y^2 + n_z^2)/(2mL^2)$

SHO: $E_n = (n + ½)(h/2\pi)\,\omega$

TRAVELING WAVE: $\psi(x) = Ae^{ikx} + Be^{-ikx}$ with $k = \dfrac{\sqrt{2mE}}{\hbar}$

DECAYING WAVE: $\psi(x) = Ae^{\alpha x} + Be^{-\alpha x}$ with $\alpha = \dfrac{\sqrt{2m(V-E)}}{\hbar}$

Rutherford Scattering: $N(\theta) = (nt/4r^2)(zZ/2K)^2 (e^2/4\pi\varepsilon_o)^2 (\sin(\theta/2))^{-4}$

Hydrogen atom: $E_n = -13.6 \text{ eV} / n^2$ $\lambda = (1/R)\, n_1^2 n_2^2/(n_1^2 - n_2^2)$ $R = 1.097 \times 10^7 \text{ m}^{-1}$

$dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$; $P(r) = r^2 |R(r)|^2$ orbital: $\boldsymbol{\mu_L} = -(e/2m)\,\boldsymbol{L}$ $\mu_{L,z} = -m_\ell \mu_B$

x-rays: $E_\alpha = 13.6 \text{ eV}\,(Z-1)^2 (3/4)$

$E = hf = h\nu = \hbar\omega$ $p = h/\lambda = \hbar k$ $k = 2\pi/\lambda$ $KE_{max} = hf - \phi$

Quantum Statistics

$P(E) = e^{-\Delta E/kT}$ $N_{BE}(E,T) = 1/[\, e^{(E-\mu)/kT} - 1\,]$ $N_{FD}(E,T) = 1/[\, e^{(E-E_{fermi})/kT} + 1\,]$

$p(E)_{MB} \sim E^{1/2} / e^{E/kT}$ $p(E)_{BE} \sim E^2/(e^{E/kT} - 1)$

$p(E)_{FD} \sim E^{1/2} / \{\exp[(E-E_F)/kT] + 1\}$ $E_F = (h^2/2m)(3N/8\pi V)^{2/3}$

$W_{N_R}^{N} = \dfrac{N!}{N_R!(N-N_R!)}$ $S = k_B \ln W$ $\partial S/\partial E = 1/T$

Nuclear

$R = (1.2 \times 10^{-15} \text{ m})A^{1/3}$ $dN/dt = -\lambda N$ $\lambda = 1/\tau = 0.693/t_{1/2}$ $K_\alpha = (A-4)Q/A$ $R = \sigma N I_o/S$

$N(t) = (R/\lambda)(1 - e^{-\lambda t})$ $\mathcal{A} = \lambda N$ (for $x + X \rightarrow y + Y$) $K_{threshold} = -Q(1 + m_x/m_X)$

(for $x + X \rightarrow A + B + C + \ldots$) $K_{threshold} = -Q(m_x + m_X + m_A + m_B + m_C + \ldots)/(2m_X)$

Mossbauer Effect: recoil energy $K = E_\gamma^2/(2Mc^2)$, $v \sim c\Delta E/E$

Particle

Leptons: e μ τ quarks [charge +2/3: u c t] [charge -1/3: d s b]

Math

$(1 \pm x)^n = 1 \pm nx + \dfrac{n(n-1)x^2}{2!} \pm \cdots$ $\ln(1+x) = x - \tfrac{1}{2}x^2 + \cdots$

$e^x = 1 + x + x^2/2! + x^3/3! + x^4/4! + \cdots$

$e^x = 1 + x + x^2/2! + x^3/3! + x^4/4! +$

$\int \sin^2(ax)dx = \dfrac{x}{2} - \dfrac{1}{4a}\sin(2ax)$ $\int \cos^2(ax)dx = \dfrac{x}{2} + \dfrac{1}{4a}\sin(2ax)$

$\int \sin(ax)\cos(ax)dx = \dfrac{1}{2a}\sin^2(ax)$ $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$