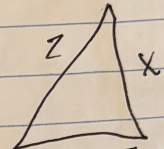


MATH | January 30th
 national crissant day

Hw question

$$\int_0^2 \frac{x^2}{\sqrt{4-x^2}} dx$$

$x = 2 \sin \theta$
 $dx = 2 \cos \theta d\theta$



$$= 2 \left[\theta - \frac{2 \sin \theta \cos \theta}{2} \right] \sqrt{4-x^2}$$

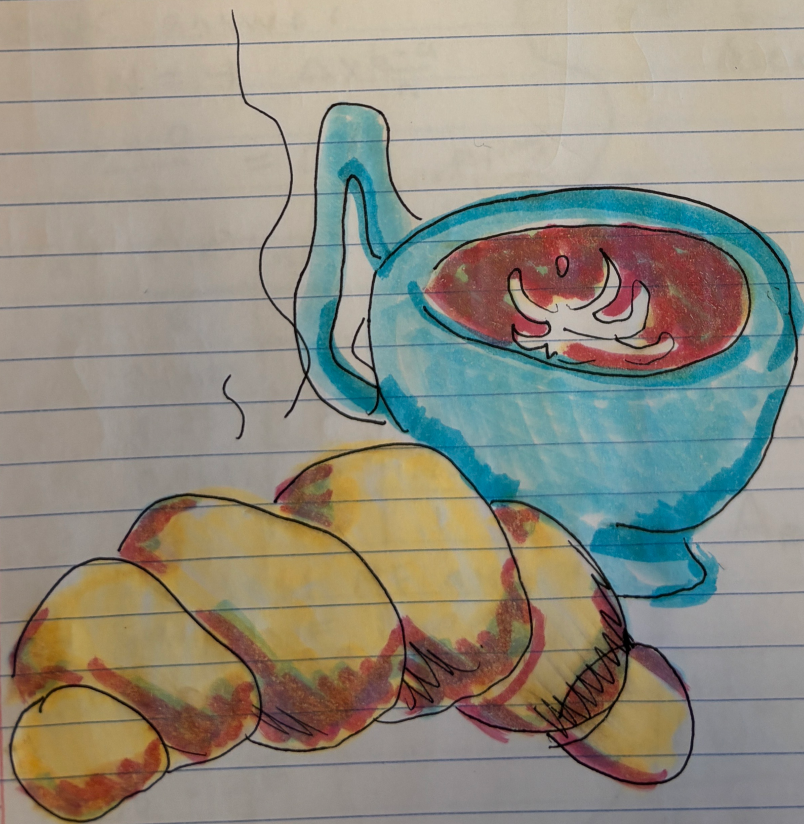
$$= \int \frac{4 \sin^2 \theta \cdot 2 \cos \theta}{\sqrt{4-4 \sin^2 \theta}}$$

$$= 4 \int \frac{1 - \cos 2\theta}{2}$$

$$= 2 \int 1 - \cos 2\theta$$

$$= 2\theta - \sin(2\theta)$$

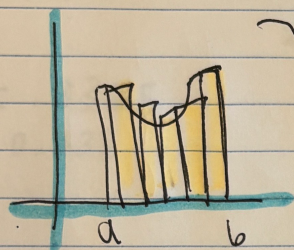
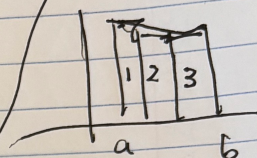
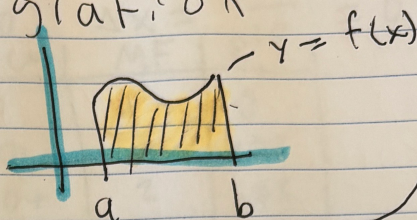
$$= 2 \left[\frac{\pi}{2} - 0 \right] = \pi$$



MATH January 30th

Numerical integration

$$\int_a^b f(x) dx =$$



improving on this

$$\int_a^b f(x) dx$$

N sub-intervals

$$\Delta x = \frac{b-a}{N}$$

we can do better than rectangles

EXP $\int_0^1 x^2 dx$

integratable so we can check answer

$$N=4 \quad \Delta x = \frac{b-a}{N}$$

$$\frac{1-0}{4} = 0.25$$

multiplying factor

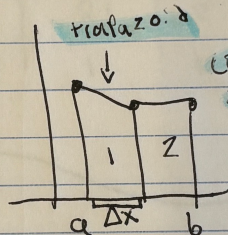
x	$f(x)$	MF	P
0	0	1	0
.25	0.0625	2	.125
.5	0.25	2	.5
.75	0.5625	2	1.25
1	1	1	1

The first and last MF are one every other MF is 2

$$\text{Sum } P \cdot \frac{\Delta x}{2}$$

$$2.75 \cdot \frac{.25}{2}$$

$$= 0.34375$$



connect the dots

A_0

Area of a trapezoid

$$\frac{H_1 + H_2}{2} \Delta x$$

$$A_0 = \frac{f(a) + f(x_1)}{2} \Delta x$$

$$A_2 = \frac{f(x_1) + f(b)}{2} \Delta x$$

$$A = \frac{\Delta x}{2} (f(a) + 2f(x_1) + f(b))$$

Trapezoid method

Example 2

$$\int_0^1 x^2 dx$$

$$\text{Sum } P = \frac{6.7 \cdot 0.1}{2}$$

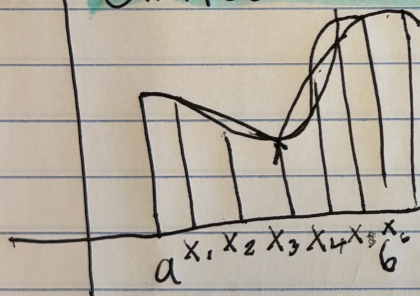
$$= .335$$

$N = 10$ $\Delta x = \frac{1-0}{10} = 0.1$

x	f(x)	MF	P
0	0	1	0
.1	.01	2	.02
.2	.04	2	.08
.3	.09	2	.18
.4	.16	2	.32
.5	.25	2	.50
.6	.36	2	.72
.7	.49	2	.98
.8	.64	2	1.28
.9	.81	2	1.62
1.0	1	1	1

Method 2

Simpson's method



$$\int_a^b f(x) dx$$

Parabola :-

you need 3 points

N must be Even

x	f(x)	MF	P
a	f(a)	1	
x ₁	f(x ₁)	4	
x ₂	f(x ₂)	2	
x ₃	f(x ₃)	4	
x ₄	f(x ₄)	2	
x ₅	f(x ₅)	4	
b	f(b)	1	

$$\sum P \cdot \frac{\Delta x}{3}$$

EXP 3 $\int_0^1 x^2 dx$ $N=10$

$$\Delta x = \frac{b-a}{N} = \frac{1-0}{10} = 0.1$$

x	f(x)	MF	P
0	0	1	0
.1	.01	4	.04
.2	.04	2	.08
.3	.09	4	.36
.4	.16	2	.32
.5	.25	4	.72
.6	.36	2	.72
.7	.49	4	1.96
.8	.64	2	1.28
.9	.81	4	3.24
1.0	1	1	1

$$10 \cdot \frac{\Delta x}{3} = \frac{1}{3} = 0.3333$$

1 at the beginning and end
alternate 2 and 4 in the middle

MATH

January 30th

Dont have to memorize

$$\text{Error}(T_N) \leq \frac{K_2(b-a)^3}{12N^2}$$

the error using trapezoid
method w n subintervals

$$K_2 = \max_{a \leq x \leq b} |f''(x)|$$

$$\text{Error}(S_N) \leq \frac{K_4(b-a)^5}{180N^4}$$

$$K_4 = \max_{a \leq x \leq b} |f''''(x)|$$