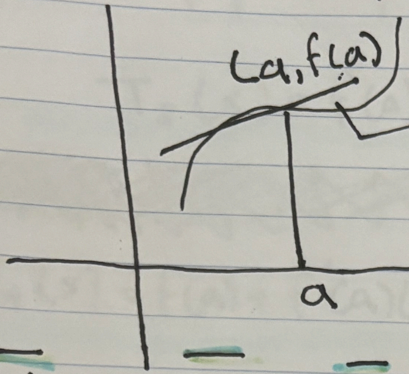
MATH

March 30th

TAYLOR Polynomials



$y = f(x)$

$y - f(a) = f'(a)(x-a)$

$y = f(a) + f'(a)(x-a)$

$L(x) = f(a) + f'(a)(x-a)$

This is the Local Linear Approximation

2nd degree Taylor Polynomial for $f(x)$ at $x = a$

$T_1(x) = f(a) + f'(a)(x-a)$

That is a 1st degree Taylor Polynomial for $f(x)$ at $x = a$

$T_2(a) = f(a)$

$T_2'(a) = f'(a)$

$T_2''(a) = f''(a)$

$T_2(x) = C_0 + C_1(x-a) + C_2(x-a)^2$

need to find 3 coefficients

$$\boxed{T_2(a) = C_0 = f(a)}$$

$T_2'(x) = C_1 + 2C_2(x-a)$

$$\boxed{T_2'(a) = C_1 = f'(a)}$$

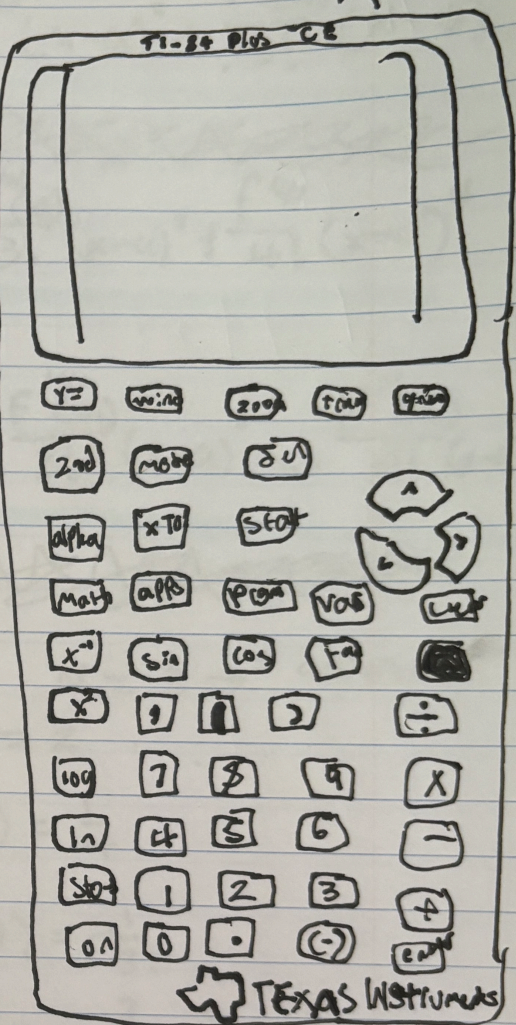
$T_2''(x) = 2C_2$

$T_2''(a) = 2C_2 = f''(a)$

$$\boxed{C_2 = \frac{f''(a)}{2}}$$

$\cancel{T_2(x) = f(a) + f'(a)(x-a)}$

$$T_2(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$$



March 30th

MATH Tylor Polynomials Pt. 2

$$T_3(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3$$

$$T_4(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \frac{f''''}{4!}(x-a)^4$$

Formula for Nth degree:

$$T_N(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f''''(a)}{3!}(x-a)^3 + \dots \frac{f^N(a)}{N!}(x-a)^N$$

EXP 1

$f(x) = \sqrt{x}$ $\quad a = 4 \quad N = 3 \rightarrow$ 3 derivitins

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$ | $f(4) = 2$

$f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}$ plug in a | $f'(4) = \frac{1}{4}$

$f''' = \frac{3}{8}x^{-\frac{5}{2}}$ | $f''(4) = -\frac{1}{32}$

$f'''(4) = \frac{3}{256}$

So

$$\boxed{T_3(x) = 2 + \frac{1}{4}(x-4) + \frac{-\frac{1}{32}}{2}(x-4)^2 + \frac{\frac{3}{256}}{3!}(x-4)^3}$$

allowed to simplify coefficients but never distribute

March 30th

MATH Taylor Polynomials pt. 3

Exp 2

$f(x) = \sin x$ at $0 = 0$ $a = 0$ $N = 7$

$f'(x) = \cos(x)$ at $0 = 1$

$f''(x) = -\sin(x)$ at $0 = 0$

$f'''(x) = -\cos(x)$ at $0 = -1$

$f''''(x) = \sin(x)$ at $0 = 0$

$f^{v}(x) = \cos(x)$ at $0 = 1$

$f^{vi}(x) = -\sin(x)$ at $0 = 0$

$f^{vii}(x) = -\cos(x)$ at $0 = -1$

if $a = 0$ we call it a MACLAULIN Polynomial ← important

$$T_7(x) = 0 + 1(x-0) + \frac{0}{2}(x-0)^2 + \frac{-1}{3!}(x-0)^3 + \frac{0}{4!}(x-0)^4 + \frac{1}{5!}(x-0)^5 + \frac{0}{6!}(x-0)^6 - \frac{1}{7!}(x-0)^7$$

$$\boxed{T_7 = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}}$$

Exp 3 $f(x) = \frac{1}{x}$ $a = 1$ $N = 3$

$f'(x) = -\frac{1}{x^2}$ $f'(1) = -1$

$f''(x) = \frac{2}{x^3}$ $f''(1) = 2$

$f'''(x) = \frac{-6}{x^4}$ $f'''(1) = -6$

$$T_3 = 1 - 1(x-1) + \frac{2}{2}(x-1)^2 + \frac{-6}{3!}(x-1)^3$$

$$\boxed{T_3 = 1 - (x-1) + (x-1)^2 - (x-1)^3}$$

How to tell How close you are to the right answer

$R_N(x)$ is the remainder

$$f(x) = T_N(x) + R_N(x)$$

$$-R_N(x) = T_N(x) - f(x)$$

error is $|R_N($

MATH

March 30th

Taylor Polynomials

$$\left| \frac{F^{N+1}(\xi)}{(N+1)!}(x-a)^{N+1} \right| = \left| R_N(x) \right|$$

ξ is between x and a