MATH

January 28th

Quiz 3

① $\int \frac{1}{x^2\sqrt{4-x^2}}\,dx$ $\quad x = 2\sin\theta \quad dx = 2\cos\theta\, d\theta$

$$\boxed{-\frac{1}{4}\,\frac{\sqrt{4-x^2}}{x} + C}$$

$$\int \frac{1}{4\sin^2\theta\,\sqrt{4(1-\sin^2\theta}}$$

$$\frac{1}{4}\int\frac{1}{\sin^2\theta} = \frac{1}{4}\int \csc^2\theta = \frac{1}{4}\cot\theta$$

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② $\int \sinh^6(3x)\cosh^3(3x)\,dx$ $\quad u = \sinh(3x) \quad du = 3\cosh(3x)\,dx$

$$dx = \frac{du}{3\cosh(3x)} \qquad \int u^6 \cosh^{\cancel{3}\,2}(3x)\,\frac{du}{3\cancel{\cosh(3x)}}$$

$$\cosh^2 x = 1 + \sinh^2 x$$

$$\frac{1}{3}\int u^6(1+u^2)\,du$$

$$\boxed{\frac{1}{3}\left(\frac{\sinh^7(3x)}{7} + \frac{\sinh^9(3x)}{9}\right) + C}$$

③ $\int \frac{5x-13}{(2x-1)(x-4)}\,dx$

$$\frac{A}{2x-1} + \frac{B}{x-4} =$$

$$\frac{A(x-4)+B(2x-1)}{(2x-1)(x-4)}$$

$x = 4$ $\qquad 5x-13 = A(x-4) + B(2x-1)$

plug in

$B = 1 \quad x = \frac{1}{2}$

$A = 3$

$$\int \frac{3}{2x-1} + \frac{1}{x-4}\,dx$$

$$\boxed{\frac{3}{2}\ln|2x-1| + \ln|x-4| + C}$$

MATH

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improper intigrals

3 cases

Case 1: ∞

ⓐ $\int_a^{\infty} f(x)\,dx$ ⓑ $\int_{-\infty}^{b} f(x)\,dx$ ⓒ $\int_{-\infty}^{\infty} f(x)\,dx$

means by definition

1a $\int_a^{\infty} f(x)\,dx \equiv \lim_{b\to\infty} \int_a^b f(x)\,dx$

I Exp $\int_1^{\infty} \frac{1}{x^2}\,dx$

way I

$= \lim_{b\to\infty} \int_1^b \frac{1}{x^2}\,dx$

$= \lim_{b\to\infty} \left(-\frac{1}{x}\right)\Big|_1^b$

$= \lim_{b\to\infty} \left(\frac{-1}{b}+1\right) = 1$

way 2

$\int_1^{\infty} \frac{1}{x^2}\,dx$

$= -\frac{1}{x}\Big|_1^{\infty}$

$= 0 - -1 = 1$

II Exp $\int_3^{\infty} \frac{1}{\sqrt{x}}\,dx$

$= 2\sqrt{x}\Big|_1^{\infty}$

$=$ DNE

Exp III

$\int_{-\infty}^{0} e^x\,dx$

$= e^x\Big|_{-\infty}^{0}$

$= 1 - 0 = 1$

Exp IV $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$

$= \arctan x\Big|_{-\infty}^{\infty}$

$= \frac{\pi}{2} - -\frac{\pi}{2} = \pi$

MATH January 28 III

improper fractions

Case 2 $\int_a^b f(x)\,dx$ where $f(a), f(b)$ or Both are not defined

EXP V $\int_0^1 \frac{1}{\sqrt{x}}\,dx$

$= 2\sqrt{x}\Big|_0^1$

$= 2\sqrt{1} - 2\sqrt{0}$

$2 - 0 = 2$

EXP VI $\int_1^2 \frac{1}{\sqrt[3]{x-2}}\,dx$

$= \frac{3(x-2)^{\frac{2}{3}}}{2}\Big|_1^2$

$0 - -\frac{3}{2} = \frac{3}{2}$

EXP VII $\int_0^1 \frac{1}{(x-1)^2}\,dx$

$= \frac{-1}{(x-1)}\Big|_0^1$

$= \cancel{-\ln|x-1|} =$ DNE

Case 3

$\int_a^b f(x)\,dx$ where $a<c<b$

$f(c)$ is not defined

$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

now u have 2 case 2

EXP VIII $\int_1^3 \frac{1}{\sqrt[3]{x-2}}\,dx$

$= \int_1^2 \frac{1}{\sqrt[3]{x-2}} + \int_2^3 \frac{1}{\sqrt[3]{x-2}}$

$= \frac{3}{2}(x-2)^{\frac{2}{3}}\Big|_1^2 + \frac{3}{2}(x-2)^{\frac{2}{3}}\Big|_2^3$

$\frac{3}{2} - 0 + 0 - \frac{3}{2} = 0$

EXP IX $\int_0^2 \frac{1}{(x-1)^2}\,dx$

$\cancel{= -\frac{1}{(x-1)}\Big|_0^2 = -1-1 = -2}$ NO!

$= \int_0^1 \frac{1}{(x-1)^2}\,dx + \int_1^2 \frac{1}{(x-1)^2}\,dy$

$= \frac{-1}{x-1}\Big|_0^1 =$ DNE