MARCH 27th Power Series

Power series look like

$$\sum_{N=1}^{\infty} a_N (x-c)^N$$

note that thiers an X

Question 1: for what values of x does the series converge

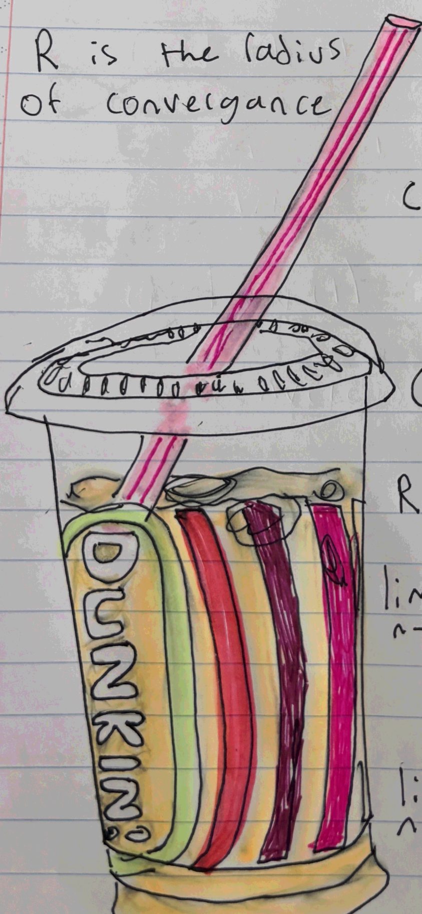
Question 2: what does it converge to?

"c is the center of the power series"

A1: Either

① converges for all x
② converges for $x = c$ only
③ converges for $|x-c| < R$

R is the radius of convergance



$c-R$ c $c+R$

$(c-R, c+R)$ interval of convergance

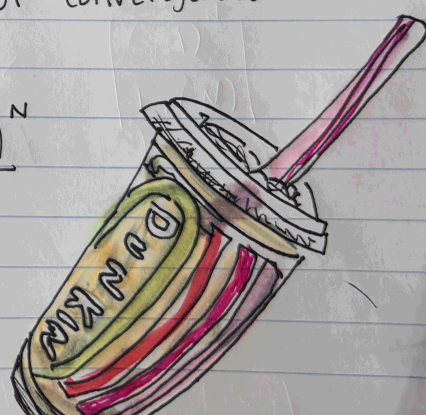
Exp

$c=2$ $$\sum_{N=1}^{\infty} \frac{(x-2)^N}{N^2 3^N}$$

Ratio test

$$\lim_{n\to\infty} \left| \frac{\dfrac{(x-2)^{N+1}}{(N+1)^2 3^{N+1}}}{\dfrac{(x-2)^N}{N^2 3^N}} \right| =$$

$$\lim_{n\to\infty} \left| \frac{(x-2)^{\cancel{N+1}}}{(N+1)^2 \cancel{3^{N+1}}} \cdot \frac{N^2 \cancel{3^N}}{\cancel{(x-2)^N}} \right| \to \left| \frac{x-2}{3} \right|$$



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Power SEries pt. 2

Exp continued

$\left|\frac{x-2}{3}\right| < 1 \rightarrow$ identical too $|x - c| < R$

$R = 3$

-1 5

2-3 2 2+3

by ratio test converges $(-1, 5)$ diverges $(-\infty, -1) \cup (5,$

What happens at -1 and 5

$X = -1 \rightarrow \sum_{n=1}^{\infty} \frac{(-1-2)^N}{N^2 3^N} \rightarrow \sum_{n=1}^{\infty} \frac{-3^N}{N^2 3^N} = \sum_{n=1}^{\infty} \frac{(-1)^N \cancel{3^N}}{N^2 \cancel{3^N}} \rightarrow \sum \frac{-1^N}{N^2}$ converg (abs)

$X = 5 \rightarrow \sum_{n=1}^{\infty} \frac{(5-2)^N}{N^2 3^N} \rightarrow \sum_{N=1}^{\infty} \frac{\cancel{3^N}}{N^2 \cancel{3^N}} \rightarrow \sum \frac{1}{N^2}$ converges p-test

$C = 2 \quad R = 3 \quad [-1, 5]$

Exp 2 $\sum_{N=0}^{\infty} \frac{(x-1)^N}{N!} \quad C = 1$

Ratio test

$\lim_{N\to\infty} \left| \frac{(X-1)^{\cancel{N+1}}}{(N+1)\cancel{!}} \cdot \frac{\cancel{N!}}{\cancel{(x-1)^N}} \right| \rightarrow \lim_{N\to\infty} \left| \frac{(x-1)}{N+1} \right| = 0 <$

all x

$R = \infty$

$(-\infty, \infty)$

c

Exp 3 $\sum_{N=1}^{\infty} \frac{N^N (x+3)^N}{5^N} \quad c = -3$

Root test $\lim_{N\to\infty} \sqrt[N]{\left| \frac{N^N (x+3)^N}{5^N} \right|} = \lim_{N\to\infty} \left| \frac{N(x+3)}{5} \right|$

$= \begin{cases} \infty \neq -3 \\ 0 = -3 < 1 \end{cases}$

converges $X = -3$ only

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Power Series Part 3

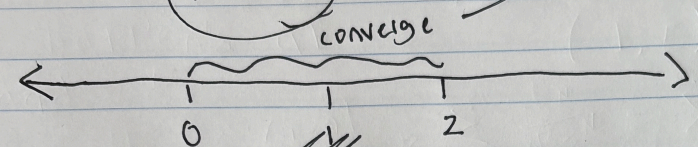
EXP 4 $\displaystyle\sum_{N=1}^{\infty} \frac{(x-1)^N}{\sqrt{N}}$ $\quad C=1$

Ratio test

$$\lim_{N\to\infty} \left| \frac{(x-1)^{\cancel{N+1}}}{\sqrt{N+1}} \cdot \frac{\sqrt{N}}{\cancel{(x-1)^N}} \right| = |x-1| < 1$$

converge

$R = 1$



$x=0 \quad \sum \frac{(-1)^N}{\sqrt{N}}$ alternating series test

$\lim_{N\to\infty} \frac{1}{\sqrt{N}} = 0$ ✓ converges condition

$\frac{1}{\sqrt{N}+1} < \frac{1}{\sqrt{N}}$ ✓

$x = 2 \quad \sum \frac{1^N}{\sqrt{N}} \to \frac{1}{\sqrt{N}} \to \frac{1}{N^{\frac{1}{2}}} \to$ diverge p-test

converges $[0, 2)$ $\quad C=1$, $R=1$

EXP 5 $\displaystyle\sum_{N=0}^{\infty} x^N$ $\quad C=0$

$R = 1$

$$\lim_{N\to\infty} \left| \frac{x^{N+1}}{x^N} \right| = |x| < 1$$

diverges at bothe end points $(-1, 1)$

x^N = Geometric Series with $R = x$

but geometric series converge when

$$\sum_{N=0}^{\infty} x^N = \frac{1}{1-x}$$

monday we start with this

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