January 26, 2026

Peanut Brittle Day

Today in History:

Australia Day (1788)

Kobe Bryant dies in helicopter crash (2020)

Number of the Day: 2519

2519 = 11 x 229

2519 is the sum of the first 41 semiprimes.

Fun Fact:

It is illegal to drive more than 2,000 sheep down Hollywood Boulevard at one time.

Quote of the Day:

"Football is unconditional love."

-Tom Brady

Today's Weather:

Variable clouds with snow showers. High 18°.

PARTIAL FRACTIONS

$$\frac{1}{3} + \frac{1}{5} = \frac{8}{15}$$

$$\frac{8}{15} \rightarrow \frac{1}{3} + \frac{1}{5}$$

DECOMMON DENOMINATORIZE

RATIONAL FUNCTIONS

$\frac{P(x)}{Q(x)}$ RULE $\underline{DEG(P(x)) < DEG(Q(x))}$

FOUR CASES

1. $Q(x)$ HAS LINEAR FACTORS $(ax+b)$
2. $Q(x)$ HAS REPEATED LINEAR FACTORS $(ax+b)^N$
3. $Q(x)$ HAS IRR. QUAD FACTORS (ax^2+bx+c)
4. ~~$Q(x)$ HAS REPEATED IRR QUAD FACTORS $(ax^2+bx+c)^N$~~

EXAMPLE $\frac{3x-1}{(x+2)(x-3)}$

$$\frac{3x-1}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3}$$

$$\frac{3x-1}{(x+2)(x-3)} = \frac{A(x-3)+B(x+2)}{(x+2)(x-3)}$$

$$\rightarrow 3x-1 = A(x-3) + B(x+2)$$

$x=3 \quad 8 = 0 + B(5) \quad B = \frac{8}{5}$

$x=-2 \quad -7 = A(-5) + 0 \quad A = \frac{7}{5}$

$$\frac{3x-1}{(x+2)(x-3)} = \frac{\frac{7}{5}}{x+2} + \frac{\frac{8}{5}}{x-3}$$

$$\int \boxed{\frac{3x-1}{(x+2)(x-3)}}\,dx = \int\left(\frac{\frac{7}{5}}{x+2} + \frac{\frac{8}{5}}{x-3}\right)dx$$

$$= \frac{7}{5}\ln|x+2| + \frac{8}{5}\ln|x-3| + C$$

EXAMPLE: $\int \frac{5x-1}{x^2-1}\,dx$

$$\frac{5x-1}{x^2-1} = \frac{5x-1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

$$\frac{5x-1}{(x-1)(x+1)} = \frac{A(x+1) + B(x-1)}{(x-1)(x+1)}$$

$$5x-1 = A(x+1) + B(x-1)$$

$\rightarrow x=1 \quad 4 = 2A \quad A = 2$

$\rightarrow x=-1 \quad -6 = -2B \quad B = 3$

$$\int \frac{5x-1}{x^2-1}\,dx = \int \frac{2}{x-1} + \frac{3}{x+1}\,dx$$

$$= 2\ln|x-1| + 3\ln|x+1| + C$$

EXAMPLE:

$$\int \frac{4x^2-3x-4}{x^3+x^2-2x}\,dx$$

$$\frac{4x^2-3x-4}{x^3+x^2-2x} = \frac{4x^2-3x-4}{x(x+2)(x-1)}$$

$$\frac{4x^2-3x-4}{x(x+2)(x-1)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-1}$$

$$\frac{4x^2-3x-4}{x(x+2)(x-1)} = \frac{A(x+2)(x-1) + Bx(x-1) + Cx(x+2)}{x(x+2)(x-1)}$$

$$4x^2-3x-4 = A(x+2)(x-1) + Bx(x-1) + Cx(x+2)$$

$x=1$ $\quad -3 = C(3) \quad C=-1$

$x=0$ $\quad -4 = -2A \quad A=2$

$x=-2$ $\quad 18 = 6B \quad B=3$

$$\int \frac{4x^2-3x-4}{x(x+2)(x-1)}\,dx = \int\left(\frac{2}{x} + \frac{3}{x+2} - \frac{1}{x-1}\right)dx$$

$$= 2\ln|x| + 3\ln|x+2| - \ln|x-1| + C$$

EXAMPLE: $\displaystyle\int \frac{3x^2-8x+13}{(x+3)(x-1)^2}\,dx$

$$\frac{3x^2-8x+13}{(x+3)(x-1)^2} = \frac{A}{x+3} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$\frac{3x^2-8x+13}{(x+3)(x-1)^2} = \frac{A(x-1)^2 + B(x+3)(x-1) + C(x+3)}{(x+3)(x-1)^2}$$

$$3x^2-8x+13 = A(x-1)^2 + B(x+3)(x-1) + C(x+3)$$

$x = 1 \qquad 8 = 4C \qquad C = 2$

$x = -3 \qquad 64 = 16A \qquad A = 4$

$x = 0 \qquad 13 = A(1) + -3B + 3C$

$13 = 4(1) - 3B + 3(2)$

$13 = 4 - 3B + 6$

$3 = -3B \qquad B = -1$

$$= \int \frac{4}{x+3} + \frac{-1}{x-1} + \frac{2}{(x-1)^2} \, dx$$

$$= 4\ln|x+3| - \ln|x-1| - \frac{2}{x-1} + C$$

EXAMPLE: $\displaystyle\int \frac{6x^2 - 3x + 1}{(4x+1)(x^2+1)} \, dx$

$$\frac{6x^2 - 3x + 1}{(4x+1)\underbrace{(x^2+1)}_{\text{IRR QUAD}}} = \frac{A}{4x+1} + \frac{Bx + C}{x^2+1}$$

$$\frac{6x^2 - 3x + 1}{(4x+1)(x^2+1)} = \frac{A(x^2+1) + (Bx+C)(4x+1)}{(4x+1)(x^2+1)}$$

$$6x^2 - 3x + 1 = A(x^2+1) + (Bx+C)(4x+1)$$

$$6x^2 - 3x + 1 = Ax^2 + A + 4Bx^2 + 4Cx + Bx + C$$

$$\boxed{6}x^2 \boxed{-3}x + \boxed{1} = \boxed{(A + 4B)}x^2 + \boxed{(4C + B)}x + \boxed{(A + C)}$$

$$A + 4B = 6$$
$$4C + B = -3$$
$$A + C = 1$$

$$A = 2$$
$$B = 1$$
$$C = -1$$

$$= \int \frac{6x^2 - 3x + 1}{(4x+1)(x^2+1)}\,dx = \int \frac{2}{4x+1} + \frac{x-1}{x^2+1}\,dx$$

$$= \int \frac{2}{4x+1} + \frac{x}{x^2+1} - \frac{1}{x^2+1}\,dx$$

$$= 2\,\frac{\ln|4x+1|}{4} + \frac{1}{2}\ln|x^2+1| - \text{ARCTAN}\,x + C$$

$$\int \frac{x^5+2}{x^2-1}\,dx$$

LONG DIVISION

$$\begin{array}{r|l} & x^3 + x \\ \hline x^2-1 & x^5 + 2 \\ & x^5 - x^3 \\ \hline & x^3 + 2 \\ & x^3 - x \\ \hline & x + 2 \end{array}$$

$$\frac{x^5+2}{x^2-1} = x^3 + x + \frac{x+2}{x^2-1}$$

$$\int \frac{x^5+2}{x^2-1}\,dx = \int x^3 + x + \frac{x+2}{x^2-1}\,dx$$

$$= \frac{x^4}{4} + \frac{x^2}{2} + \boxed{\int \frac{x+2}{x^2-1}\,dx}$$