Math

Feb 26th

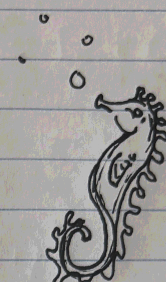
quiz recap

① $T_0 = 70$

$Y(10) = 150 \longrightarrow Y(0) = 150$

$Y(15) = 134 \qquad Y(5) = 134$

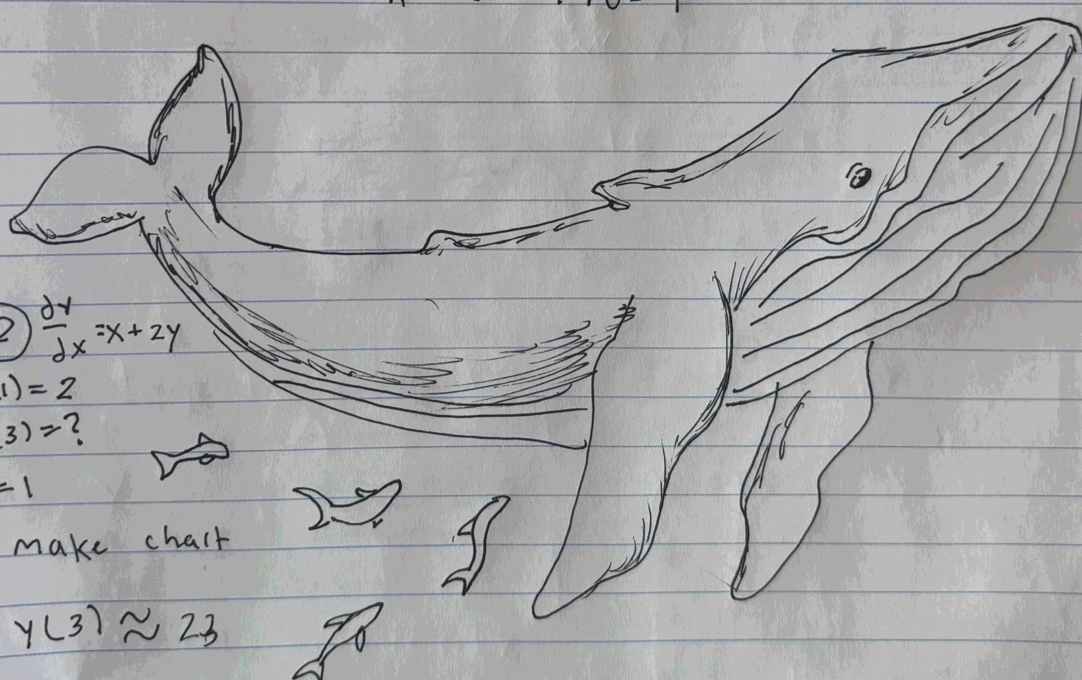
$Y(-10) = ?$



$$\frac{dy}{dt} = -k(y - T_0)$$

$y = 70 + Ce^{-kt}$ $\qquad C = 80$

$k = 0.044629$



② $\frac{dy}{dx} = x + 2y$

$y(1) = 2$

$y(3) = ?$

$h = 1$

make chart

$y(3) \approx 23$

③ $CC = 1000 \qquad Y(1) = 200 \qquad Y(4) = 500 \qquad Y(0) = ?$

$\downarrow$

$Y(0) = 200 \qquad Y(3) = 500$

$A = 1000$

$K = 0.462$

$Y_0 = 200$

$B = -\frac{1}{4} \qquad \frac{200}{200-1000}$

$A\ -\frac{1}{4}$

$$y = \frac{1000}{1 + 4e^{-kt}}$$

$$\boxed{Y(-1) = 136}$$

Math

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First order linear differential equations (F.O.L.D.E)

First order: only has first derivitive
linear: y and y' are to the 1st power

Standard form $y' + P(x)y = Q(x)$

Integrating factor: $\alpha x = e^{\int P(x)dx}$

multiply to both sides → $\alpha x y' + \alpha x P(x) y = \alpha x Q(x)$

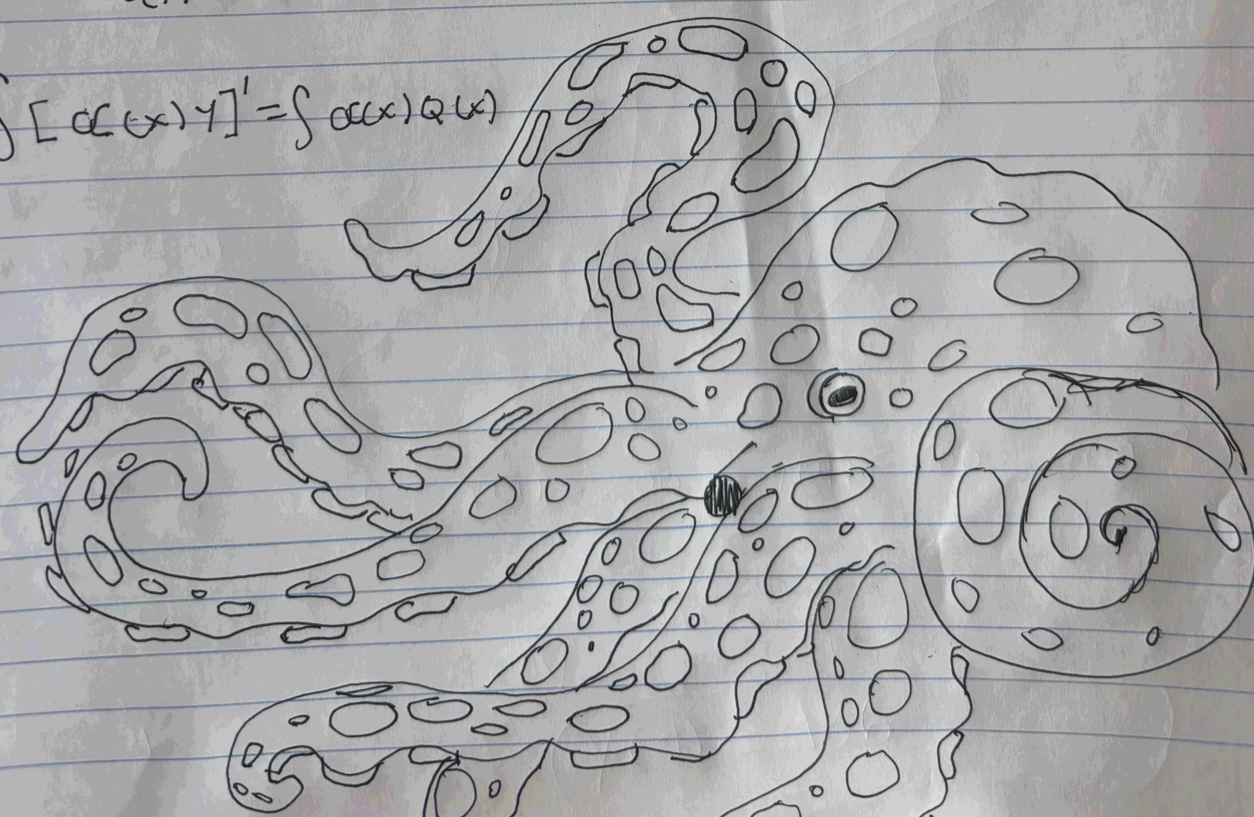
$$[\alpha(x)y]' = \alpha x Q(x)$$

bc

$$[\alpha(x)y]' = \alpha(x)y' + y\alpha'(x)$$

$\alpha(x) = e^{\int P(x)dx}$ $\alpha(x)' = e^{\int P(x)dx}$ $P(x) = \alpha(x)P(x)$

$$\int[\alpha(x)y]' = \int \alpha(x)Q(x)$$



Math

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wall to stop you from killing

$$y = \frac{1}{\alpha(x)}\left[\int \alpha(x)\,Q(x)\,dx + C\right]$$

Exp 1 $\quad y' + \frac{3}{x}y = 6x^2$

$P(x) = \frac{3}{x}$

$Q(x) = 6x^2$

yay ̈

$\alpha(x) = e^{\int \frac{3}{x}dx} = e^{3\ln x} = x^3$

$y = \frac{1}{\alpha(x)}\left[\int \alpha(x)\,Q(x)\,dx + c\right]$

$y = \frac{1}{x^3}\left[x^6 + c\right]$

Exp 2 $\quad y' + 4xy = x$

$y' = x - 4xy$

$P(x) = 4x$

$Q(x) = x$

$\alpha(x) = e^{\int 4x\,dx} = e^{2x^2}$

$y = \frac{1}{\alpha x}\left[\int \alpha x\, Q(x)\,dx + c\right] = \frac{1}{e^{2x^2}}\left[\int x e^{2x^2}dx + c\right]$

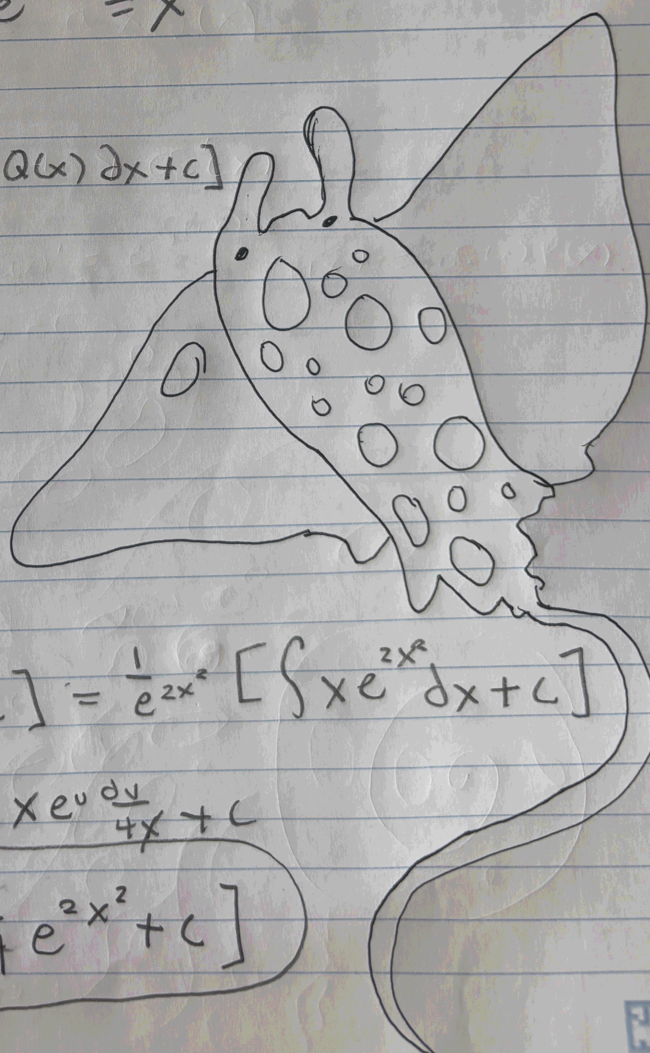
$u = 2x^2$

$du = 4x\,dx$

$\frac{du}{4x} = dx$

$y = \frac{1}{e^{2x^2}}\left[\int x e^{u}\frac{du}{4x} + c\right]$

$e^{-2x^2}\left[\frac{1}{4}e^{2x^2} + c\right]$



MAth feb 26th

Exp 3 $x^2y' + xy = 1$ $y(1) = 2$

$P(x) = \frac{1}{x}$ $y' + \frac{1}{x}y = \frac{1}{x^2}$

$Q(x) = \frac{1}{x^2}$

$\alpha x = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

$y = \frac{1}{x}\left[\int \cancel{x}\frac{1}{\cancel{x^2}}dx + C\right)$

$y = \frac{1}{x}\left[\int \frac{1}{x}dx + c\right]$

$y = \frac{1}{x}\left[\ln|x| + 2\right]$ — Plug in initial condition

$2 = \frac{1}{1}[\ln 1 + c]$

$2 = \cancel{\ln(1)} + c$

$c = 2$

Exp 4 ̈

$y' = e^2x + 3y$

$y(0) = 3$

$y' - 3y = e^2x$

$P(x) = -3$

$Q(x) = e^2$

$\alpha(x) = e^{\int -3dx} \Rightarrow e^{-3x}$

$y = \frac{1}{e^{-3x}}\left[\int e^{-3x+2x}dx + c\right]$

$y = e^{3x}\left[-e^{-x} + C\right]$

$3 = C - 1$

$y = e^{3x}\left[-e^{-x} + 4\right]$