MATH

march 23rd

HW review

MPod 10: $\sum_{N=1}^{\infty} \frac{N}{\sqrt{N^3+2N}}$

$$\frac{N}{\sqrt{N^3+2N}} \sim \frac{N}{\sqrt{N^3}} = \frac{N}{N^{3/2}} \to \frac{1}{\sqrt{N}}$$

$$\frac{N}{\sqrt{N^3+2N}} < \frac{1}{N^{1/2}} \qquad \sum \frac{1}{N^{1/2}} \text{ diverges (p test)} \to \text{LCT}$$

$$\lim_{n\to\infty} \frac{\frac{N}{\sqrt{N^3+2N}}}{\frac{1}{\sqrt{N}}} \to \lim_{N\to\infty} \frac{N^{3/2}}{\sqrt{N^3+2N}} = 1 \to \boxed{\text{diverges}}$$

$\sum_{N=2}^{\infty} \frac{1}{\ln(N)}$ $\quad$ $\ln(N) < N$ so $\frac{1}{\ln(N)} > \frac{1}{N}$

since $\sum \frac{1}{N}$ diverges by LCT

$\frac{1}{\ln(n)}$ diverges

RATIO test / if I were a geometric series what kind of geometric series would I be tes

$\sum_{n=1}^{\infty} a_n \qquad a_n > 0$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = R \begin{cases} R<1 \to \text{series converges} \\ R>1 \to \text{series diverges} \\ R=1 \to \text{no information} \end{cases}$$

RATIO test Examples

Exp 1 $\sum_{n=1}^{\infty} \frac{2^N}{N!}$

$\lim_{n\to\infty} \frac{a_{N+1}}{a_N} = \lim_{n\to\infty} \dfrac{\frac{2^{N+1}}{(N+1)!}}{} \longrightarrow \lim_{n\to\infty} \frac{2^{\cancel{n+1}}}{(n+1)!} \frac{n!}{\cancel{2^n}}$

becomes $\lim_{n\to\infty} \frac{2}{n+1} = 0$ $\quad \frac{2^N}{N!}$ by Ratio test $\boxed{\text{converges}}$

Exp 2 $\sum_{N=1}^{\infty} \frac{N^2 2^{N+1}}{3^N}$ $\quad \lim_{N\to\infty} \frac{a_{N+1}}{a_N} = \lim_{N\to\infty} \dfrac{\frac{(N+1)^2 2^{N+2}}{3^{N+1}}}{\frac{N^2 2^{N+1}}{3^N}}$

$\lim_{N\to\infty} \frac{(N+1)^2 2^{\cancel{N+2}}}{3^{\cancel{N+1}}} \frac{\cancel{3^N}}{N^2 \cancel{2^{N+1}}} = \frac{2}{3}$

by R.T $\boxed{\text{converges}}$

Exp 3 $\sum_{N=1}^{\infty} \frac{2^N}{N}$ $\quad \lim_{N\to\infty} \frac{a_{N+1}}{a_N}$

$\lim_{N\to\infty} \dfrac{\frac{2^{N+1}}{N+1}}{\frac{2^N}{N}} = \lim_{N\to\infty} \frac{2\cancel{2^{N+1}}}{N+1} \frac{N}{\cancel{2^N}} = 2$

by R.T $\boxed{\text{diverges}}$

Exp 4 $\sum_{N=1}^{\infty} \frac{\sqrt{N}}{N^2+1}$ $\quad \lim_{N\to\infty} \dfrac{\frac{\sqrt{N+1}}{(N+1)^2+1}}{\frac{\sqrt{N}}{N^2+1}}$

can't tell ratio test inconclusive

$\lim_{N\to\infty} \frac{\sqrt{N+1}}{(N+1)^2+1} \frac{N^2+1}{\sqrt{N}} = 1$

Exp 5 $\sum_{N=1}^{\infty} \frac{(N!)^2}{(2N)!}$ $\quad \lim_{N\to\infty} \dfrac{\frac{((N+1)!)^2}{(2N+1)!}}{\frac{(N!)^2}{(2N)!}} \longrightarrow \frac{(N+1)!^2}{2(N+1)!} \frac{2N!}{(N!)^2}$

$\lim_{N\to\infty} \frac{(N+1)^2}{(2N+1)(2N+2)} = \frac{1}{4} < 1$ $\boxed{\text{converges}}$

ROOT test / soon to be your favorite

$$\sum_{N=1}^{\infty} a_N \qquad a_N > 0$$

$$\lim_{N \to \infty} \sqrt[N]{a_N} = R \begin{cases} R < 1 & \text{series conv} \\ R > 1 & \text{series diverge} \\ R = 1 & \text{no info} \end{cases}$$

Exp Root test

exp 1 $\displaystyle\sum_{N=1}^{\infty} \left(\frac{N}{2N+1}\right)^N \qquad \lim_{N \to \infty} \sqrt[N]{\left(\frac{N}{2N+1}\right)^{\cancel{N}}} \to \lim_{N \to \infty} \frac{N}{2N+1}$

$= \frac{1}{2}$ series $\boxed{\text{converges}}$

Exp 2 $\displaystyle\sum_{N=1}^{\infty} \left(\frac{3N^2+4N+5}{N^2+1}\right)^N \qquad \lim_{N \to \infty} \sqrt[N]{\left(\frac{3N^2+4N+5}{N^2+1}\right)^{\cancel{N}}}$

$$\lim_{N \to \infty} \frac{3N^2+4N+5}{N^2+1} = 3 \quad \boxed{\text{series diverges}}$$

Exp 3 $\displaystyle\sum_{N=1}^{\infty} \left(1 + \frac{1}{N}\right)^N \qquad \lim_{N \to \infty} \cancel{\sqrt[N]{}}\left(1+\frac{1}{N}\right)^{\cancel{N}} = \lim_{N \to \infty} 1 + \frac{1}{N}$

$\to 1$ $\boxed{\text{inconclusive}}$

---

5 tests when to use which

$\displaystyle\sum_{N=1}^{\infty} a_N$ $\boxed{\text{N-th term}} \xrightarrow{=0}$

$\downarrow \neq 0$ $\boxed{\text{diverges}}$

test	good	bad
intigral	ln N	$N!$ 2^N
comparison test	$\frac{N \text{ powers}}{N \text{ powers}}$	cant compare
L.C.T	$\frac{N \text{ powers}}{N \text{ powers}}$	sin N cos N
RATIO	$N!$. 2^N	$\frac{N \text{ powers}}{N \text{ powers}}$
ROOT	$(\quad)^N$	anything else