Math Pt. 1 January 23rd

Hyperbolic trig substitution

Recall

$\sinh x = \frac{e^x - e^{-x}}{2}$	$\cosh x \frac{e^x + e^{-x}}{2}$
$\tanh x = \frac{\sinh x}{\cosh x}$	$\coth x = \frac{\cosh x}{\sinh x}$
$\sec x = \frac{1}{\cosh x}$	$\operatorname{csch} x = \frac{1}{\sinh x}$

$$\cosh^2 x - \sinh^2 x = 1$$

Identities

$\sinh^2 x = \frac{\cosh 2x - 1}{2}$
$\cosh^2 x = \frac{\cosh 2x + 1}{2}$
$\cosh^2 x - \sinh^2 x = 1$

$\frac{d}{dx} \sinh x = \cosh x$	$\frac{d}{dx} \cosh x = \sinh x$
$\frac{d}{dx} (\tanh x) = \operatorname{sech}^2 x$	negative $\frac{d}{dx} \coth x = -\operatorname{csch}^2 x$
negative $\frac{d}{dx} \operatorname{sech} x = -\operatorname{sech} \tanh$	negative $\frac{d}{dx} \operatorname{csch} x = -\operatorname{csch} x \coth x$

$$\int \sinh x \, dx = \cosh x + C$$

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Part II

(A) $\int \sinh^m x \cosh^N x \, dx$

1.) if m is odd $u = \cosh x$

2.) if N is odd $u = \sinh x$

3.) Both Even Use identities

Exp $\int \sinh^3 x \cosh^2 x \, dx$ $\quad u = \cosh x$

$du = \sinh x \, dx$

$= \int \cancel{\sinh}^3 x \, u^2 \frac{du}{\cancel{\sinh x}}$ $\quad dx = \frac{du}{\sinh x dx}$

$= \sinh^2 x \, u^2 \, du$ $\quad \int (u^2 - 1) u^2 \, du = \int u^4 - u^2 \, du$

$\frac{u^5}{5} - \frac{u^3}{3} = \frac{\cosh^5 x}{5} - \frac{\cosh^3 x}{3} + C$

(B) $\int \tanh^m x \operatorname{sech}^N x \, dx$

(1) N Even $U = \tanh x$

(2) m odd $U =$ ~~tanh~~ $\operatorname{sech} x$

(3) N Odd m Even ~~…~~ Run

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Part III

(c) $\int \coth^m x \, \operatorname{csch}^n x \, dx$ (same as B)

Inverse HyperBolic trig functions

$y = \operatorname{Arcsinh} x$ $\qquad \sinh y = x$

$\cosh^2 y - \sinh^2 y = 1$

$\frac{d}{dx}(\operatorname{Arcsinh} x)$ $\qquad \frac{d}{dx}(\sinh y = x) \to (\cosh y)\frac{dy}{dx} = 1$ implicitly diferentiate

$\cosh y \, \cancel{\sinh y} \, \frac{dy}{dx} = 1$ $\qquad \frac{dy}{dx} = \frac{1}{\cancel{\sinh y} \, \cosh y}$

$\sinh^2 y = \cos^2 y - 1$ $\qquad \sinh y = x$

$\cosh^2 y = 1 + \sinh^2 y \to 1 + x^2$ $\qquad \cosh y = \sqrt{1+x^2}$

$$\boxed{\frac{1}{\sqrt{1+x^2}} = \frac{dy}{dx} \operatorname{Arcsinh} x}$$

Exp $\int \frac{1}{\sqrt{4+x^2}} dx$ $\qquad x = 2\sinh\theta$, $\quad dx = 2\cosh\theta \, d\theta$

$$\int \frac{2\cosh\theta \, d\theta}{\sqrt{4 + 4\sinh^2\theta}} = \int \frac{2\cosh\theta}{\sqrt{4(1+\sinh^2\theta)}}$$

$$\int \frac{2\cosh\theta}{\sqrt{4\cosh^2\theta}} d\theta \to \frac{\cancel{2\cosh\theta}}{2\cosh\theta} d\theta$$

$$\int 1 \, d\theta = \theta = \operatorname{Arcsinh}\left(\frac{x}{2}\right) + C$$

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Part IV

$y = \operatorname{Arccosh} x$

$\frac{\partial}{\partial x}(\cosh y = x)$

$\downarrow$

$\sinh y \frac{dy}{dx} \Rightarrow 1$

$\rightarrow \frac{dy}{dx} = \frac{1}{\sinh y}$

$\sqrt{\sinh^2 y = x^2 - 1}$

$\sinh y = \sqrt{x^2-1}$

$\frac{dy}{dx} = \frac{1}{\sqrt{x^2-1}}$

Example $\int \frac{1}{\sqrt{x^2-9}}\, dx$

$x = 3\cosh\theta$

$dx = 3\sinh\theta\, d\theta$

$\int \frac{3\sinh\theta\, d\theta}{\sqrt{9\cosh^2\theta - 9}} = \int \frac{3\sinh\theta}{\sqrt{9(\cosh^2\theta - 1)}}\, d\theta = \int \frac{3\sinh\theta}{3\sinh\theta}\, d\theta$

$$\boxed{\operatorname{Arccosh}\left(\frac{x}{3}\right) = \operatorname{Arccosh} x \frac{dy}{dx}}$$

$y = \operatorname{Arctanh} x \quad \frac{\partial}{\partial x}(\tanh y = x)$

$(\operatorname{sech}^2 y)\frac{dy}{dx} = 1 \qquad \frac{dy}{dx} = \frac{1}{\operatorname{sech}^2 y}$

$\cosh^2 y - \sinh^2 y = 1 \rightarrow 1 - \tanh^2 y = \operatorname{sech}^2 y$

$$= \frac{1}{1-\tanh^2 y} = \boxed{\frac{1}{1-x^2} = \operatorname{Arctanh} x \frac{dy}{dx}}$$