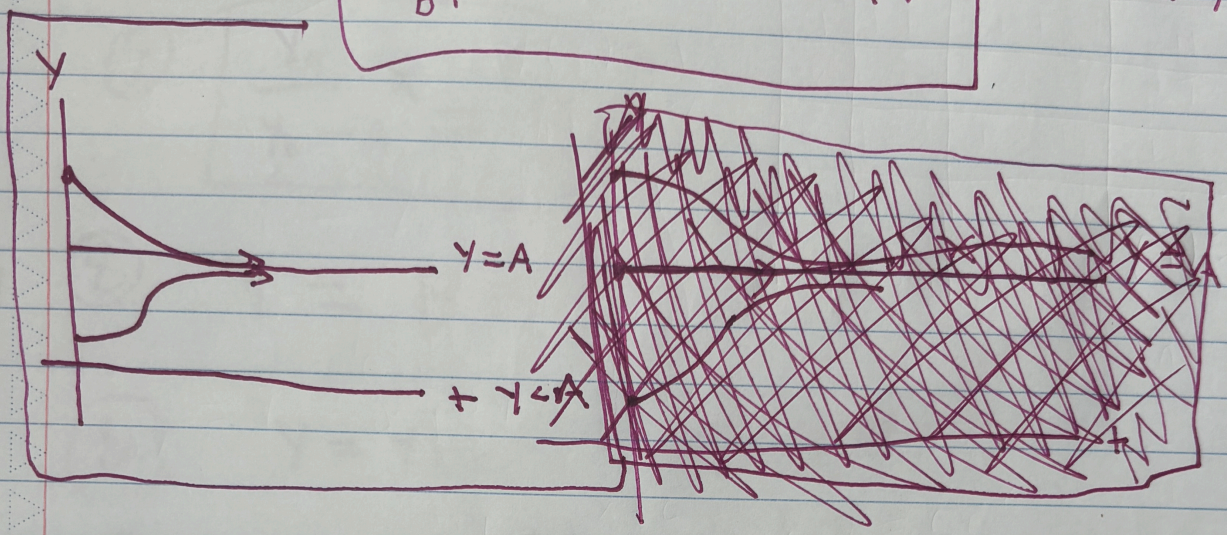
MATH: Logistic growth

$$\frac{dy}{dt} = Ky\left(1 - \frac{y}{A}\right)$$

A is the carrying capacity



① $\frac{dy}{dt} = Ky\left(1 - \frac{y}{A}\right)$ $\quad y(0) = y_0$ initial condition

② $\frac{1}{y\left(1 - \frac{y}{A}\right)}\, dy = K\, dt$

③ $\int \frac{1}{y\left(1 - \frac{y}{A}\right)}\, dy = \int K\, dt = Kt + C$

④ $\int \frac{1}{y\left(1 - \frac{y}{A}\right)} dy$ $\quad \frac{1}{y\left(1 - \frac{y}{A}\right)} = \frac{E}{y} + \frac{F}{1 - \frac{y}{A}}$

④.1 $E\left(1 - \frac{y}{A}\right) + FY = 1$

④.2 $y = 0$ $\quad 1 = E$ $\qquad y = A$ $\quad 1 = FA$ $\quad F = \frac{1}{A}$

④.3 $\int \frac{1}{y} + \frac{\frac{1}{A}}{1 - \frac{y}{A}}\, dy = \int \frac{1}{y} + \frac{1}{A - y}\, dy$

⑤ $\ln y - \ln(y - A) = \ln_e\left(\frac{y}{y - A}\right) = kt + C$

MAth: logistic growth

(6) $\frac{Y}{Y-A} = Ce^{kt}$

(7) $\frac{Y_0}{Y_0 - A} = B$

olve for Y

(8) $Y = BeY^{kt} - AY$

(8.1) $Y = YBe^{kt} - ABe^{kt}$

(8.2) $Y - YBe^{kt} = -ABe^{kt}$

(8.3) $Y(1 - Be^{kt}) = -ABe^{kt}$

(8.4) $Y = \frac{-ABe^{kt}}{1 - Be^{kt}}$

(9) $Y = \frac{ABe^{kt}}{Be^{kt} - 1} =$

$$Y = \frac{A}{1 - \frac{e^{-kt}}{B}} \qquad B = \frac{Y_0}{Y_0 - A}$$

Just use the formula it will be provided on quizes and tests

MATh: logistic growth

Exp 1.) $\frac{dy}{dt} = 0.4y\left(1 - \frac{y}{9}\right)$ $\quad y(0) = 3$, $y(2) = ?$

A, K, y_0, B
all unknowns

$A = 9$ (look at equation)

$K = 0.4$

$y_0 = 3$ — initial condition

$$B = \frac{y_0}{y_0 - A} = \frac{3}{3-9} = \frac{3}{-6} = -\frac{1}{2}$$

$$y = \frac{A}{1 - \frac{e^{-kt}}{B}} = \frac{9}{1 - \frac{e^{-0.4t}}{-\frac{1}{2}}}$$

$$= \frac{9}{1 + 2e^{-0.4t}}$$

↘ check to see if this makes sense by plugging in initial condition

$$y(2) = \frac{9}{1 + 2e^{-0.4(2)}} = \boxed{4.74}$$

MATH: logistic growth

calling capacity

Exp 2: sheep

$y = \frac{A}{1 - \frac{e^{kt}}{B}}$

$cc = 200$ $\quad A = 200$

$k = 0.2/y_1$ $\quad k = 0.2$

$y(0) = 150$ $\quad y_0 = 150$

$y(1) = ?$ $\quad B = \frac{y_0}{y_0 - A} = 3$

$y(1) = \frac{200}{1 + \frac{e^{-0.2}}{3}} = 157.12$ ← rounded to whole number

Exp 3: Rabbits

$y(0) = 1000$

$y(1) = 1050$

$y(2) = 2500$

$A = 5000$

$k = ?$

$y_0 = 1000$

$B = \frac{y_0}{y_0 - A} = -0.25$ $\quad CC = 5000$

$y = \frac{A}{1 - \frac{e^{-kt}}{B}} = \frac{5000}{1 - \frac{e^{-kt}}{-0.25}} = \frac{5000}{1 + 4e^{-kt}}$

Plug in $y(1) = 1050$

$\frac{5000}{1 + 4e^{-k}} = 1050$, $k = 0.0614$

$y = \frac{5000}{1 + 4e^{-0.614t}}$

$2500 = \frac{5000}{1 + 4e^{-0.614t}}$

$t = 22.57$ yrs

MAth: logistic growth

Exp 4: turtles

$A = 400$ $\quad K = 0.8$ $\quad cc = 400$

$K = 0.8$

$y_0 = ? = 500$ $\quad y(1) = 500$ $\quad y(0) = ?$

$B = \frac{y_0}{y_0 - 400} = ? = 5$ (make $y_0 = y(1)$)

$\frac{400}{1 - e^{(0.8)\frac{1}{5}}}$

$= 721$

600