MATH: April 22

Cross Product

AKA vector products

$\vec{a} = \langle a_1, a_2, a_3 \rangle$

$\vec{a} \times \vec{b}$

$\vec{b} = \langle b_1, b_2, b_3 \rangle$

$$\boxed{\vec{a} \times \vec{b} = \det \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}}$$

$$\begin{matrix} \vec{i} & \vec{j} \\ a_1 & a_2 \\ b_1 & b_2 \end{matrix}$$

$$\boxed{\vec{a} \times \vec{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle}$$

Exp 1

$$\begin{matrix} & \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ \vec{a} = & \langle 3, & 4, & 5 \rangle & 3 & 4 \\ \vec{b} = & \langle 8, & 7, & 6 \rangle & 8 & 7 \end{matrix}$$

$\vec{a} \times \vec{b} = \langle -11, 22, -11 \rangle$

The result of a cross product is always a vector

vectors have magnitude and direction

$\vec{a} \cdot (\vec{a} \times \vec{b}) = \langle a_1, a_2, a_3 \rangle \cdot \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$

$a_1(a_2 b_3 - a_3 b_2) + a_2(a_3 b_1 - a_1 b_3) + a_3(a_1 b_2 - a_2 b_1)$

$\cancel{a_1 a_2 b_3} - \cancel{a_1 a_3 b_2} + \cancel{a_2 a_3 b_1} - \cancel{a_2 a_1 b_3} + \cancel{a_3 a_1 b_2} - \cancel{a_3 a_2 b_1}$

$\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$

MATH

April 22

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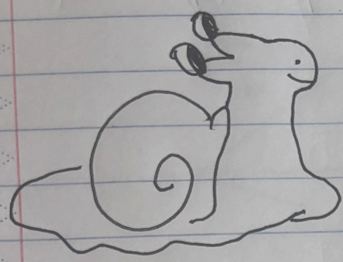


$\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$

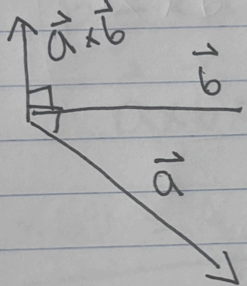
and

$\vec{b} (\vec{a} \times \vec{b}) = 0$

that means $\vec{a}$ is perpendicular to $\vec{a} \times \vec{b}$ and

$\vec{b}$ is perpendicular to $\vec{a} \times \vec{b}$

use the right hand rule to verify direction



Magnitude: $\|\vec{a} \times \vec{b}\|$

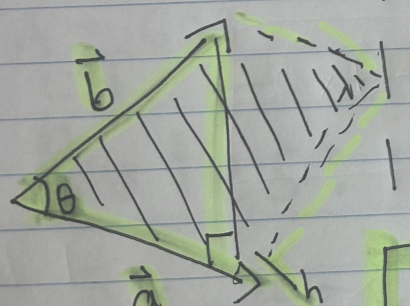
$$\|\vec{a} \times \vec{b}\|^2 = \|\vec{a}\|^2 \|\vec{b}\|^2 - (\vec{a} \cdot \vec{b})^2$$

$$= \|\vec{a}\|^2 \|\vec{b}\|^2 - (\|\vec{a}\| \|\vec{b}\| \cos\theta)^2$$

$$= \|\vec{a}\|^2 \|\vec{b}\|^2 - \|\vec{a}\|^2 \|\vec{b}\|^2 \cos^2\theta$$

$$= \|\vec{a}\|^2 \|\vec{b}\|^2 (1 - \cos^2\theta)$$

$$= \|\vec{a}\|^2 \|\vec{b}\|^2 (\sin^2\theta)$$



$$\boxed{\|\vec{a} \times \vec{b}\| = \|\vec{a}\| \|\vec{b}\| \sin\theta}$$

$A_{\square} = \|\vec{a}\| h$

equal

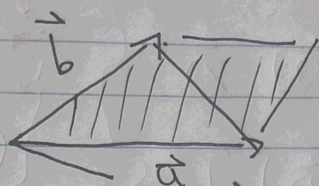
$h = \|\vec{b}\| \sin\theta$

$$\boxed{A_{\square} = \|\vec{b}\| \|\vec{a}\| \sin\theta}$$

MATH: Cross Product

April 22

$\boxed{A_\Delta = \frac{1}{2}\|\vec{a} \times \vec{b}\|}$



area of triangle

$\vec{i}$ $\vec{j}$ $\vec{k}$ $\vec{i}$ $\vec{j}$

Exp $\vec{a} = \langle 2, 5, 1 \rangle$ 2 5

$\vec{b} = \langle 6, 4, 3 \rangle$ 6 4

$\vec{a} \times \vec{b} = \langle 11, 0, -22 \rangle$

$\|\vec{a} \times b\| = A$

$= \sqrt{11^2 + 0^2 + (-22)^2}$

$A = \sqrt{605}$

You can verify answer by knowing its got to be parallel to $\vec{a}$ and $\vec{b}$ so

$\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$

check that, check

$\vec{b} \cdot (\vec{a} \times \vec{b}) = 0$

Rules:

- dot (·) product checks if two vectors are perpendicular
- cross (×) product ~~checks~~ creates a vector thats perpendicular to two others

1. $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$
2. $\vec{a} \times \vec{a} = \vec{0}$ ← zero vector
3. $\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})$
4. $\vec{a} = k\vec{b} \rightarrow \vec{a} \times \vec{b} = \vec{0}$