Math Part I January 21st

Trig Substitution

3 cases

Ⓐ $\sqrt{a^2 - x^2}$ $x = a\sin\theta$

Exp $\int \frac{1}{\sqrt{4-x^2}} dx$ $x = 2\sin\theta$ $dx = 2\cos\theta$

$\int \frac{2\cos\theta}{\sqrt{4(1-\sin^2\theta)}} d\theta$

$\int \frac{2\cos\theta}{\sqrt{4\cos^2\theta}} d\theta \int \frac{2\cos\theta}{2\cos\theta} d\theta$

$\int 1\, d\theta$ $x = 2\sin\theta$

$\frac{x}{2} = \sin\theta$

$\theta = \arcsin\left(\frac{x}{2}\right)$

Quiz 2

① $\int \sqrt{x} \ln x$

$u = \ln x$ $dv = \sqrt{x}\, dy$

$du = \frac{1}{x} dx$ $v = \frac{2}{3} x^{3/2}$

$\frac{2}{3} x^{3/2} \ln x - \int \frac{1}{\cancel{x}} \frac{2}{3} x^{\cancel{3/2}\ \frac{1}{2}}$

$\frac{2}{3} x^{\frac{3}{2}} \ln x - \frac{4}{9} x^{3/2} + C$

② $\int x^2 e^x dx$ $u = x^2$ $dv = e^x dx$

$x^2 e^x - \int 2x e^x dx$ $du = 2x dx$ $v = e^x$

$u = 2x$ $dv = e^x dx$

$du = 2$ $v = e^x$ $x^2 e^x - [2xe^x - \int 2e^x dx]$

$x^2 e^x - 2xe^x + 2e^x + C$

③ $\int \frac{\sin^2 x}{\sin^3 x} = \int \sin^2 x \cos^{3} x\, dx$ $u = \sin x$ $du = \cos dx$

$\int u^2 \cos^{\cancel{3}\,2} \cdot \frac{du}{\cos} = \int u^2 (1-u^2) du =$

$\int (u^2 - u^4) du = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$

④ $\int \tan^3 x \sec^3 x\, dx$

$u = \sec$ $\int \tan^{\cancel{3}\,2} u^{\cancel{3}\,2} \frac{du}{\sec\tan}$

$du = \sec\tan dx$

$\int (u^2 - 1) u^2 du \to u^4 - u^2 du$

$\frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$

Part II

Math

January 21st

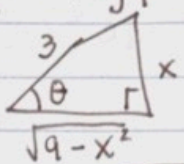
Exp $\int \frac{1}{x^2\sqrt{9-x^2}}\,dx$ $x = 3\sin\theta$

$dx = 3\cos\theta\, d\theta$

factor out 9 1- $\sin^2 = \cos^2$

$= \int \frac{\cancel{3}\cos\theta}{9\sin^2\theta\sqrt{9-\cancel{9\sin^2\theta}}}\,d\theta \rightarrow \int \frac{1}{9\sin^2\theta}\,d\theta \rightarrow \int \frac{1}{9}\csc^2\theta\, d\theta$

$\rightarrow -\frac{1}{9}\cot\theta'$ $x=3\sin\theta$ $\frac{x}{3}\sin\theta$



$\boxed{-\frac{1}{9}\frac{\sqrt{9-x^2}}{x}+C}$

Ⓑ $\sqrt{a^2+x^2}$ o $\sqrt{x^2+a^2}$ $x = a\tan\theta$

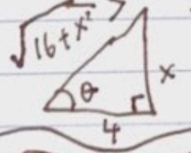
Exp $\int \frac{1}{\sqrt{16+x^2}}$ $x = 4\tan\theta$

$dx = 4\sec^2\theta\, d\theta$

$= \int \frac{4\sec^2\theta}{\sqrt{16+16\tan^2\theta}}\,d\theta \rightarrow \int \frac{4\sec^2\theta}{\sqrt{16\sec^2\theta}}\,d\theta = \int \frac{4\sec^2\theta}{4\sec\theta}\,d\theta$

$\int \sec\theta\, d\theta \longrightarrow \ln|\sec\theta+\tan\theta|$ On the formula sheet!!

$\tan\theta = \frac{x}{4}$



$= \boxed{\ln\left|\frac{\sqrt{16+x^2}}{4}+\frac{x}{4}\right|+C}$

Exp $\int \frac{1}{(25+x^2)^{3/2}}$ $x = 5\tan\theta$

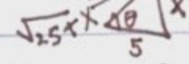
$dx = 5\sec^2\theta\, d\theta$

$\int \frac{1}{(\sqrt{25+x^2})^3}$ $\frac{5\sec^2\theta}{(\sqrt{25+25\tan^2\theta})}\,d\theta$

$\cancel{\int \frac{5\sec^2\theta}{\sqrt{25}}}$ $\int \frac{5\sec^2\theta}{(5\sec\theta)^3}\,d\theta$

$\int \frac{\cancel{5}\sec^2\theta}{\cancel{5}\,25\sec^3\theta}\,d\theta \rightarrow \frac{1}{25}\int \cos\theta\, d\theta$

$\frac{1}{25}\int \cos\theta\, d\theta$



$\tan\theta = \frac{x}{5}$

↓

$\boxed{\frac{1}{25}\frac{x}{\sqrt{25+x^2}}+C}$

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Part III

Math January 21st

ⓒ $\sqrt{x^2-a^2}$ $x = a\sec\theta$

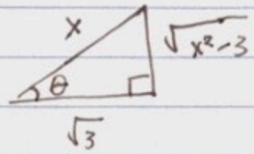
Exp $\int \frac{\sqrt{x^2-3}}{x}$ $a = \sqrt{3}$ $\rightarrow \int \frac{\sqrt{3\sec^2\theta - 3}}{\cancel{\sqrt{3}}\,\cancel{\sec\theta}}$ $\cancel{\sqrt{3}}\,\cancel{\sec\theta}$ tanθ d

$x = \sqrt{3}\sec\theta$

$dx = \sqrt{3}\sec\theta\tan\theta\, d\theta$

$$\int \sqrt{3(\sec^2\theta - 1)}\tan\theta\, d\theta = \int \sqrt{3(\tan^2\theta}\, d\theta = \int \sqrt{3}\tan^2 d\theta$$

$$\sqrt{3}\int (\sec^2\theta - 1)\, d\theta = \sqrt{3}(\tan\theta - \theta)$$



$$\boxed{\sqrt{3}(\frac{\sqrt{x^2-3}}{\sqrt{3}} - \text{ArcSec}(\frac{x}{\sqrt{3}}) + c}\ \frac{x}{\sqrt{3}} = \sec\theta$$

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