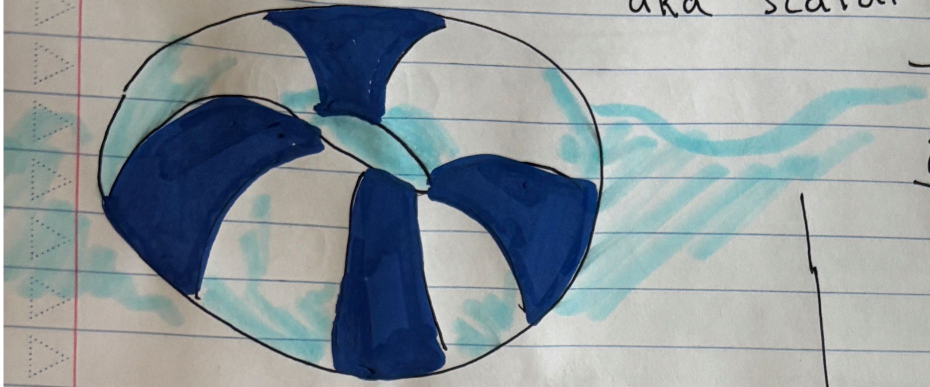
MATH

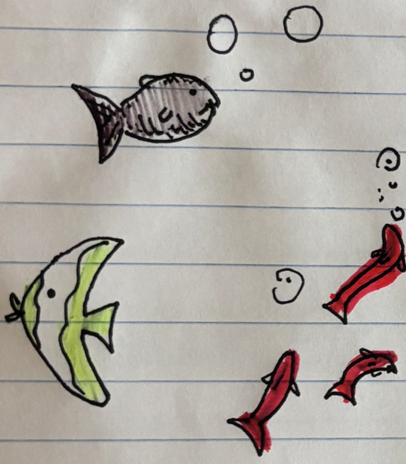
April 20th

Dot Product

aka scalar Product



$$\vec{a} = \langle a_1, a_2, a_3 \rangle$$
$$\vec{b} = \langle b_1, b_2, b_3 \rangle$$
$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$



Exp: $\vec{a} = \langle 3, 5, 4 \rangle$

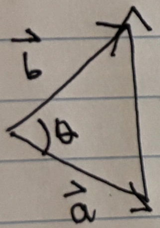
$\vec{b} = \langle 1, 2, 3 \rangle$

$\vec{a} \cdot \vec{b} = \overset{3+10+12}{25} = 3(1) + 5(2) + 4(3)$

Physics formula: boo 0/10 bad

$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos\theta$$

Law of cosines

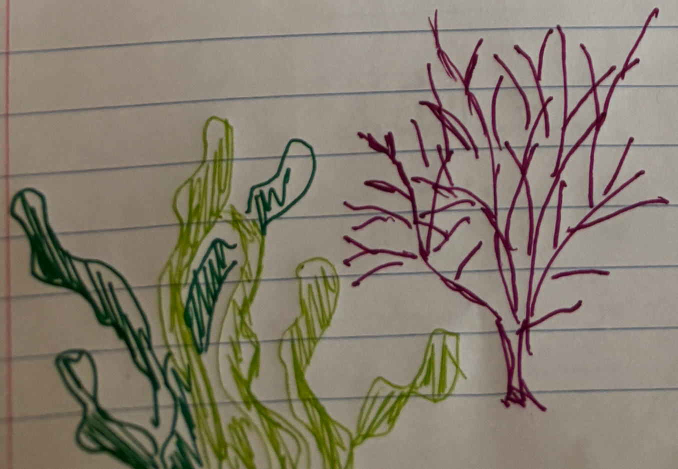


$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$\vec{c} = \vec{a} - \vec{b}$

$$\|\vec{c}\|^2 = \|\vec{a}\|^2 + \|\vec{b}\|^2 - 2\|\vec{a}\| \|\vec{b}\| \cos\theta$$

$$\|\langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle\|^2 = \|\langle a_1, a_2, a_3\|^2$$
$$+$$
$$\|\langle b_1, b_2, b_3 \rangle\|$$
$$-$$
$$2\|\vec{a}\| \|\vec{b}\| \cos\theta$$



April 20th

MATH: dot product

$$\left[\left(\sqrt{(a_1-b_1)^2+(a_2-b_2)^2+(a_3-b_3)^2}\right)^2=\left(\sqrt{a_1^2+a_2^2+a_3^2}\right)^2+\left(\sqrt{b_1^2+b_2^2+b_3^2}\right)\ldots\right.$$
$$\left.\ldots-2\|\vec{a}\|\|\vec{b}\|\cos\theta\right]$$

$$\left[\cancel{a_1^2}-2a_1b_1+\cancel{b_1^2}+\cancel{a_2^2}-2a_2b_2+\cancel{b_2^2}+\cancel{a_3^2}-2a_3b_3+\cancel{b_3^2}\ldots\right.$$
$$\left.=\cancel{a_1^2}+\cancel{a_2^2}+\cancel{a_3^2}+\cancel{b_1^2}+\cancel{b_2^2}+\cancel{b_3^2}-2\|\vec{a}\|\|\vec{b}\|\cos\theta\right]$$

$$\boxed{\begin{gathered}a_1b_1+a_2b_2+a_3b_3=\|\vec{a}\|\|\vec{b}\|\cos\theta\\ \cos\theta=\frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\|\vec{b}\|}\end{gathered}}$$

~~Ex~~ Exp $\vec{a}=\langle 3,5,4\rangle$ $\quad\cos\theta=\frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\|\vec{b}\|}$

$\vec{b}=\langle 1,2,3\rangle$

$\|a\|=\sqrt{9+25+16}=\sqrt{50}$ $\quad\cos\theta=\frac{25}{\sqrt{50}\sqrt{14}}$

$\|b\|=\sqrt{1+4+9}=\sqrt{14}$

Exp $\vec{a}=\langle -1,3,2\rangle$ $\quad\vec{a}\cdot\vec{b}=\overset{-6+0+6}{0}$

$\vec{b}=\langle 6,0,3\rangle$ $\quad\cos\theta=0=\frac{0}{\sqrt{14}\sqrt{\ }}$

$\|\vec{a}\|=\sqrt{1+9+4}=\sqrt{14}$

$\|\vec{b}\|=\sqrt{36+0+9}=\sqrt{45}$

This means the vectors are Perpendicular

MATH

April 20th

IF $\vec{a} \cdot \vec{b} = 0 \iff \vec{a} \perp \vec{b}$

IF $\vec{a} \cdot \vec{b} > 0 \Rightarrow \cos\theta > 0$

$0 < \theta \leq \frac{\pi}{2}$ θ is acute

IF $\vec{a} \cdot \vec{b} < 0 \Rightarrow \cos\theta < 0$

$\frac{\pi}{2} < \theta$ θ is obtuse

Rules:

① $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

② $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

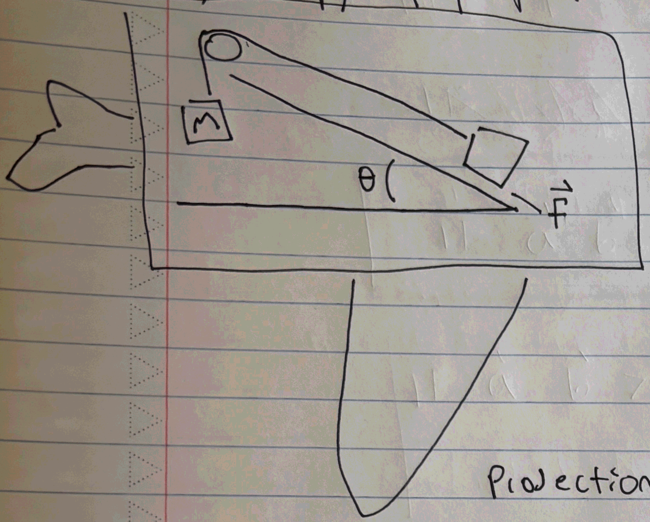
③ $k(\vec{a} \cdot \vec{b}) = k\vec{a} \cdot \vec{b} = \vec{a} \cdot k\vec{b}$

④ $\vec{a} \cdot \vec{a} = a_1, a_1 + a_2, a_2 + a_3 a_3 = \|\vec{a}\|^2$

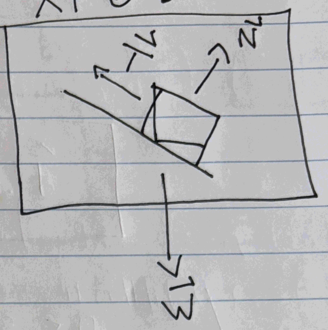
MATH

April 20th

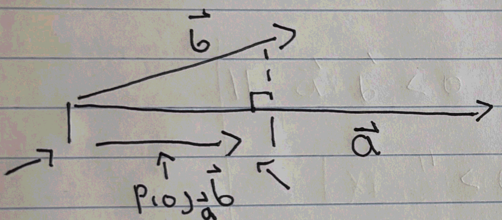
Dot Product



XFBD



Projection of $\vec b$ in direction $\vec a$ ($\text{Proj}_{\vec a}\vec b$)



$$\|\text{Proj}_{\vec a}\vec b\| = \|\vec b\|\cos\theta$$

$$= \cancel{\|\vec b\|}\,\frac{\vec a\cdot\vec b}{\|\vec a\|\cancel{\|\vec b\|}}$$

$$\boxed{\|\text{Proj}_{\vec a}\vec b\| = \frac{|\vec a\cdot\vec b|}{\|\vec a\|}}$$

we know this is positive

$\text{Proj}_{\vec a}\vec b$ can be written as $\text{comp}_{\vec a}\vec b$

$$\boxed{\text{Proj}_{\vec a}\vec b = \frac{\vec a\cdot\vec b}{\|\vec a\|}\left(\frac{\vec a}{\|\vec a\|}\right)}$$

Exp:

$\vec a = \langle 3,5,4\rangle$

$\vec b = \langle 1,2,3\rangle$

unit vector

$$\|\text{Proj}_{\vec a}\vec b\| = \frac{|\vec a\cdot\vec b|}{\|\vec a\|}$$

$$= \frac{25}{\sqrt{50}}$$

$$\text{Proj}_{\vec a}\vec b = \frac{25}{\sqrt{50}}\cdot\frac{\langle 3,5,4\rangle}{\sqrt{50}}$$

$\text{Proj}_{\perp\vec a}\vec b$

MATH

April 20th

Dot Product

$$\text{Proj}_{\vec{a}}\vec{b} + \text{Proj}_{\vec{a}\perp}\vec{b} = \vec{b}$$

$$\boxed{\text{Proj}_{\perp\vec{a}}\vec{b} = \vec{b} - \text{Proj}_{\vec{a}}\vec{b}}$$