Comparison & Limit comp

Remember...

$$\sum_{N=1}^{\infty} \frac{1}{N^3+1}$$ ... need a new test

comparison Test / Big Brother Test

$\sum_{n=1}^{\infty} a_N \quad \sum_{n=1}^{\infty} b_N \quad a_N, b_N > 0$

Then if $a_N < b_N$ $\quad \sum_{N=1}^{\infty} b_N$ converges $\Rightarrow \sum_{N=1}^{\infty} a_N$ converge

If $a_N < b_N$ $\quad \sum_{N=1}^{\infty} a_N$ diverges $\Rightarrow \sum_{N=1}^{\infty} b_N$ diverges

story: b_N is like big sibling

if b_N has to stay in, a_N does too

Back to:

$\sum_{N=1}^{\infty} \frac{1}{N^3+1} \quad \frac{1}{N^3+1}$ is like $\frac{1}{N^3}$ is bigger

we know $\frac{1}{N^3}$ converges by p-test $\quad p=3>1$

so $\frac{1}{N^3+1}$ also converges

Example #2

$\sum_{N=1}^{\infty} \frac{1}{3N-2} \quad \frac{1}{3N-2} > \frac{1}{3N} \quad \sum_{N=1}^{\infty} \frac{1}{3N} = \frac{1}{3}\sum_{N=1}^{\infty} \frac{1}{N}$ Diverges b/c Harmonic

diverge

Example #3

$\sum_{N=1}^{\infty} \frac{5^N-2}{7^N+3^N} \quad \frac{5^N-2}{7^N+3^N} < \frac{5^N}{7^N} \quad \sum_{N=1}^{\infty} \frac{5^N}{7^N} = \sum_{N=1}^{\infty} \left(\frac{5}{7}\right)^N$ converge b/c geometric $\frac{5}{7} < 1 = R$

converge

Example #4

← b/c sin is never > 1

$\sum_{N=1}^{\infty} \frac{\sin^2 N}{N^2}$ $\qquad \frac{\sin^2 N}{N^2} < \frac{1}{N^2}$ $\qquad \sum_{N=1}^{\infty} \frac{1}{N^2}$ converges $P = 2 > 1$

↳ converges

Example #5

$\sum_{N=2}^{\infty} \frac{1}{N^2 - N}$ $\qquad \frac{1}{N^2-N} > \frac{1}{N^2}$ $\qquad \sum_{N=1}^{\infty} \frac{1}{N^2}$ converges $P = 2 > 1$ $\qquad \cancel{a_N \le b_N}$

↳ need a new test

Limit comparison test / L.C.T. / UP / Hooking up

$\sum_{N=1}^{\infty} a_N, \sum_{N=1}^{\infty} b_N \quad a_N, b_N > 0$

$\lim_{n \to \infty} \frac{a_N}{b_N} = L$ if $0 < L < \infty$

Then either both $\sum a_N, \sum b_N$ conv. or both div.

idea: the man & woman in the movie UP started looking like each other as they grew older

Limit test is like compatibility test

Example #1

$\sum_{N=2}^{\infty} \frac{1}{N^2 - N}$ $\qquad \frac{1}{N^2 - N} \sim \frac{1}{N^2}$

$\lim_{n \to \infty} \frac{\frac{1}{N^2-N}}{\frac{1}{N^2}} = \lim_{n \to \infty} \frac{N^2}{N^2 - N} = 1 \qquad 0 < 1 < \infty$

Since $\sum \frac{1}{N^2}$ converges $\Rightarrow \sum_{N=2}^{\infty} \frac{1}{N^2-N}$ converges

Example #2

$\sum_{N=1}^{\infty} \frac{1}{3N+2}$ $\qquad \frac{1}{3N+2} \sim \frac{1}{3N}$

$\lim_{n \to \infty} \frac{\frac{1}{3N}}{\frac{1}{3N+2}} = \lim_{n \to \infty} \frac{3N+2}{3N} = 1 \qquad 0 < 1 < \infty$

$\sum \frac{1}{3N} = \frac{1}{3} \sum \frac{1}{N} \to$ diverges

Since $\sum \frac{1}{3N}$ diverges, $\sum_{N=1}^{\infty} \frac{1}{3N+2}$ diverges

Example #3

$\sum_{N=1}^{\infty} \frac{1}{\sqrt{N^2 + 2N + 5}}$ $\qquad \frac{1}{\sqrt{N^2+2N+5}} \sim \frac{1}{\sqrt{N^2}} = \frac{1}{N}$ $\qquad \sum \frac{1}{N}$ Diverges Harmonic

L.C.T $\lim_{n \to \infty} \frac{\frac{1}{\sqrt{N^2+2N+5}}}{\frac{1}{N}} = \lim_{n \to \infty} \frac{N}{\sqrt{N^2+2N+5}} = 1$

$\therefore$ Both Diverge

Example #4

$$\sum_{n=1}^{\infty} \frac{5^n+6^n}{7^n-2^n} \qquad \frac{5^n+6^n}{7^n-2^n} \sim \frac{6^n}{7^n}$$

L.C.T $\lim_{n\to\infty} \dfrac{\frac{6^n}{7^n}}{\frac{5^n+6^n}{7^n-2^n}} = \lim_{n\to\infty} \dfrac{6^n}{7^n} \dfrac{7^n-2^n}{5^n+6^n} = \lim_{n\to\infty} \dfrac{\frac{7^n-2^n}{7^n}}{\frac{5^n+6^n}{6^n}}$

$= \lim_{n\to\infty} \dfrac{\overset{1}{\cancel{\frac{7^n}{7^n}}} - \overset{\to 0}{\frac{2^n}{7^n}}}{\underset{\to 0}{\frac{5^n}{6^n}} + \cancel{\frac{6^n}{6^n}}1} = 1$ $\quad \sum \frac{6^n}{7^n} = \sum \left(\frac{6}{7}\right)^n$ converges

$\Rightarrow \sum \frac{5^n+6^n}{7^n-2^n}$ converges