Math:

March 18th

quiz recap

① $\left\{\frac{7N^3+8N}{9+5N-4N^2-2N^3}\right\}_{N=1}^{\infty} = -\frac{7}{2}$

② $\left\{\sqrt{N^2+3N}-N\right\}_{N=1}^{\infty}$ Multiply by conjagate

$\lim\limits_{n\to\infty} \sqrt{N^2+3N}-N\left(\frac{\sqrt{N^2+3N}+N}{\sqrt{N^2+3N}+N}\right) = \frac{3}{2}$

③ $\sum_{N=1}^{\infty} \frac{7N^3+8N}{9+5N-4N^2-2N^3}$

Nth term test

$\lim\limits_{n\to\infty} = -\frac{7}{2} \rightarrow$ Diverges

④ $\sum_{N=0}^{\infty} 2^{2N}5^{1-N} = \sum_{N=0}^{\infty} \frac{5 \cdot 4^N}{5^N} = \sum 5\left(\frac{4}{5}\right)^N$

$R = \frac{4}{5}$

$|R| < 1$ converges

$\frac{a}{1-R} = \frac{5}{1-\frac{4}{5}} = 25$

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Series tests

Series Recap

$$\sum_{N=a}^{\infty} a_N = a_1 + a_2 + a_3 + a_4 + \dots$$

Definition $S_j = \sum_{N=1}^{j} a_N$ $\lim_{j \to \infty} S_j = S$

Series converges and has sum S

- Geometric Series

$$\sum_{N=0}^{\infty} aR^N \begin{cases} \text{Div} & |R| \geq 1 \\ \text{conv } \frac{a}{1-R} & |R| < 1 \end{cases}$$

- Harmonic Series

$$\sum_{N=1}^{\infty} \frac{1}{N} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$ Diverges!!

- Telescoping

expand series out and cancel terms

- N^{th} term / divergence / george test

$$\lim_{n \to \infty} a_N \begin{cases} = 0 & \text{? can't tell} \\ \neq 0 & \text{diverges} \end{cases}$$

New test: Intigral Test.

$a_N > 0$ $\sum_{N=1}^{\infty} a_N$ If $a_N = f(N)$ for all N and $f(x)$ is cont. and decreasing

Then if $\int_1^{\infty} f(x)\,dx$ converges (a Number)

Then $\sum a_N$ conv.

if $\int_1^{\infty} f(x)$ Div $\to$ $\sum a_N$ div

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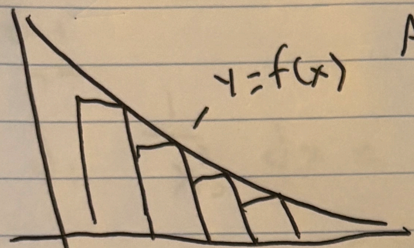
Proving intigral test

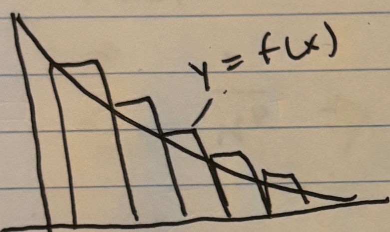


$A_{curve} > A_{boxes}$

$$\int_1^\infty f(x)\,dx < f(1)+f(2)+\dots$$

$$< a_1 + a_2 + a_3 + \dots$$

$$< \sum_{N=1}^{\infty} a_N$$



$A_{curve} < A_{boxes}$

Exp 1

$$\sum_{N=0}^{\infty} \frac{1}{1+N^2} \to f(x) = \frac{1}{1+x^2}$$

f(x) ☑ cont. ☑ dec

$$\int_0^\infty \frac{1}{1+x^2}\,dx = \arctan x \Big|_0^\infty = \frac{\pi}{2} \text{ (number)}$$

Converges not at $\frac{\pi}{2}$ tho

Exp 2

$$\sum_{N=1}^{\infty} \frac{N}{N^2+1} \qquad f(x) = \frac{x}{x^2+1}$$

☑ cont. ☑ dec.

$$\int_1^\infty \frac{x}{x^2+1}\,dx \qquad u = x^2+1 \quad \frac{du}{2x} = dx \qquad = \frac{1}{2} \ln|x^2+1| \Big|_1^\infty$$

= Diverges

Exp 3

$$\sum_{N=1}^{\infty} \frac{1}{\sqrt{N}} \qquad f(x) = \frac{1}{\sqrt{x}}$$

cont ✓ dec ✓

$$\int_1^\infty \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x} \Big|_1^\infty = \text{Diverge}$$

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EXP 4

$$\sum_{N=1}^{\infty} \frac{1}{N^3} \qquad f(x) = \frac{1}{x^3}$$

☑ cont.
☑ Dec

$$\int_1^{\infty} \frac{1}{x^3}\, dx = -\frac{1}{2x^2}\Big|_1^{\infty} = \frac{1}{2}$$

$\boxed{\text{Converges not at } \tfrac{1}{2} \text{ necessarily}}$

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p - test

$$\sum_{N=1}^{\infty} \frac{1}{N^p} \Rightarrow \begin{cases} p>1 & \text{converges} \\ p\leq 1 & \text{diverges} \end{cases}$$