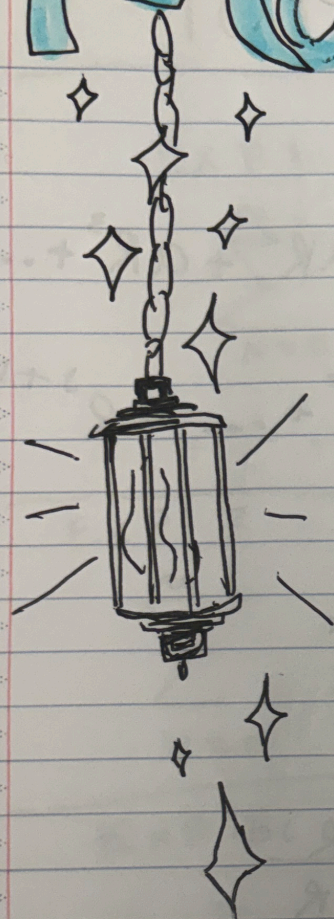
math: Series

march 16th



A sequence is an ordered list of numbers

$a_1, a_2, a_3, a_4, a_5 + \dots$

the only question is what is the limit

A Series

$$a_1 + a_2 + a_3 + a_4 + a_5 + (\dots) = \sum_{N=1}^{\infty} a_N$$

↑ Problem

Exp

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \dots$$

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots$$

← sequence of partial sums

$$\sum_{N=1}^{\infty} a_N \qquad S_j = \sum_{N=1}^{j} S_N = a_1 + a_2 + a_3 + \dots a_j$$

$$S_1 = a_1 \qquad S_2 = a_1 + a_2 \qquad S_3 = a_1 + a_2 + a_3 \dots$$

$$\lim_{j \to \infty} S_j = S$$

the sum converges with some S

converges → has a sum

march 16th

Math Series

Geometric series $\sum_{N=0}^{\infty} aR^N$

$$S_j = \sum_{N=0}^{j} aR^N = a + aR + aR^2 + aR^3 + \dots aR^j$$

$$RS_j = R\sum_{N=0}^{\infty} aR^N = aR + aR^2 + \dots aR^{j+1}$$

$$S_j - RS_j = a - aR^{j+1}$$

$$S_j(1-R) = (a - aR^{j+1}) \text{ or}$$

$$S_j = \frac{a - aR^{j+1}}{1-R}$$

$$\lim_{j\to\infty} S_j = \lim_{j\to\infty} \frac{a - aR^{j+1}}{1-R}$$

$$\boxed{\lim_{j\to\infty} \frac{a - aR^{j+1}}{1-R} \begin{cases} \text{Div} & |R| \geq 1 \\ \frac{a}{1-R} & |R| > 1 \end{cases}}$$

Math Series March 16th

Exp 1

$$\sum_{N=0}^{\infty} 3\left(\frac{1}{2}\right)^N \qquad a = 3$$

$$R = \frac{1}{2} < 1$$

$$\frac{a}{1-R} = \frac{3}{1-\frac{1}{2}} = 6$$

Exp 2

$$\sum_{N=0}^{\infty} 2\left(-\frac{2}{3}\right)^N \qquad a = 2 \qquad R = -\frac{2}{3} \quad |R| < 1$$

$$\frac{a}{1-R} = \frac{2}{1+\frac{2}{3}} = \frac{6}{5}$$

Exp 3

$$\sum_{N=0}^{\infty} -\frac{1}{2}(7)^N \qquad a = -\frac{1}{2}$$

$$R = 7 \geq 1 \quad \text{Div}$$

Math series

March 16th

Harmonic Series

$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \dots = S$

$\frac{1}{2} S = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} + \dots$

$S - \frac{1}{2} S = \frac{1}{2} S = 1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \dots$

$\cancel{\tfrac{1}{2}} S > \cancel{\tfrac{1}{2}} S$???

$S > S$

$\boxed{\text{Harmonic Series Diverge!!!}}$

but it diverges super slowly

N th term test aka divergent test aka George

$\sum_{N=1}^{\infty} a_N \quad \lim_{N \to \infty} a_N = \begin{cases} = 0 \quad \text{[redacted]} \\ \neq 0 \quad \text{Series diverges} \end{cases}$

Exp 3 $\sum_{N=1}^{\infty} \frac{2N^2 + 3N + 5}{6N^2 + 2N + 4}$ $\lim_{N \to \infty} \frac{2N^2 + 3N + 5}{6N^2 + 2N + 4} = \frac{1}{3} \neq$

diverges

Exp 4 $\sum_{N=1}^{\infty} \frac{1}{N(N+1)}$ $\lim_{N \to \infty} \frac{1}{N(N+1)} = 0$

if $\lim_{N \to \infty} a_N = 0$ you dont know whether it converges or not march 16th

Math Series

Telescoping

$$\sum_{N=1}^{\infty}\left(\frac{1}{N}-\frac{1}{N+1}\right)=\left(1-\cancel{\tfrac{1}{2}}\right)+\left(\cancel{\tfrac{1}{2}}-\cancel{\tfrac{1}{3}}\right)+\left(\cancel{\tfrac{1}{3}}-\cancel{\tfrac{1}{4}}\right)+\ldots$$

$$=1$$

Expand then shrink you know the sum as long as the last term goes to 0