MATh

diferential equations (D.E)

for the next 2 weeks
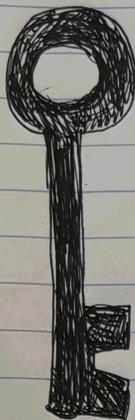
what is a differential equation?

an equation with a derivative in it

EXPs

- $\frac{dy}{dx} = 3x + 4y$

- $y'' + 3y' + 2y = 0$

What is a Solution to a D.E?

find a function that makes the equation true

How do we find a Solution?

EXP:

D.E fairy

① $\frac{dy}{dx} = 3y^{\frac{2}{3}}$ ⟨$y = x^3$⟩ ✓

$3x^2 \stackrel{?}{=} 3(x^3)^{\frac{2}{3}}$ $\frac{dy}{dx} = 3x^2$

② $y'' + 4y = 0$ ⟨$y = \sin x$⟩ ✗
    $y' = \cos x$
    $y'' = -\sin x$

$-\sin x + 4\sin x = 0$ NO

DNE!!!

⟨$y = \sin(2x)$⟩ ✓ $y' = 2\cos 2x$
    $y'' = -4\sin(2x)$

$-4\sin(2x) + 4\sin(2x) = 0$

How to Salve w/o fairy

⇨ Seperable D.E.

Step 1 $\frac{dy}{dx} = \frac{f(x)}{g(y)}$ 1st order, has 1st derivitive

2nd order ≡ has 2nd derivative

So on and so forth

Step 2 $\int g(y)\,dy = \int f(x)\,dx$

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feb 16th

differential equations (D.E)

EXP   Seperable D.E

1.) $(x^2 + 4)\frac{dy}{dx} = xy$

2.) $\int \frac{1}{y} dy = \int \frac{x}{x^2+4} dx$

3.) $\ln(y) = \frac{1}{2}\ln(x^2+4) + C$

4.) $y = e^{\frac{1}{2}\ln(x^2+4) + C}$ $e^{\ln y}$ $e^{\frac{1}{2}\ln(x^2+4)} \cdot e^C$

only on one Side be it represents both unknown constants combined

5.) $y = e^{\ln(x^2+4)^{\frac{1}{2}}}$ $\downarrow$ C

$\boxed{y = C\sqrt{x^2+4}}$
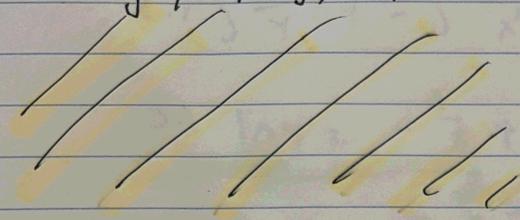
EXP 2 seperable D.E

C is adaptable

1.) $y' = \frac{x^2}{y^2}$

2.) $y^2 dy = x^2 dx$

3.) $\int y^2 dy = \int x^2 dx$

4.) $\frac{y^3}{3} = \frac{x^3}{3} + C$

5.) $y^3 = x^3 + C$

6.) $y = \sqrt[3]{x+C}$

$\frac{dy}{dx} = f(x)$
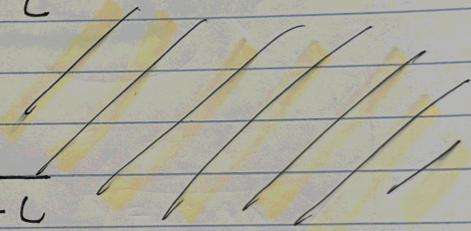
1.) $\frac{dy}{dx} = f(x)$

2.) $\int dy = \int f(x) dx$

$y = \int f(x) dx$

we have been doing diffrential equations

How can you tell a D.E is seperable. if you can seperate it   no special trick

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EXP 3 seperable D.E

1.) $\frac{dy}{dx} = \frac{6x^2}{2y + \cos y}$

2.) $\int (2y + \cos y) dy = \int 6x^2 dx$

3.) $\boxed{y + \sin y = 2x^3 + C}$

can't solve for y

EXP 4 Seperable D.E

1.) $\frac{dy}{dx} = \frac{xy-y}{y+1}$

2.) $\frac{dy}{dx} = \frac{y(x-1)}{y+1}$

3.) $\int \frac{y+1}{y} dx = \int (x-1) dx$

4.) $y + \ln y = \frac{x^2}{2} - x + C$

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Differential Equations (D.E)

Expt: initial condition $\rightarrow y(1) = 3$

1.) $\frac{dy}{dy} = \frac{y}{x^2}$

2.) $\int \frac{1}{y} dy = \int \frac{1}{x^2} dx$

3.) $\ln y = -\frac{1}{x} + c$

6.) $3 = ce^{-\frac{1}{1}} = 3 = ce^{-1}$

7.) $c = 3e$

4.) $y = e^{-\frac{1}{x}} e^{c}$

5i) $y = ce^{-\frac{1}{x}}$

General solution

$\downarrow$

Particular Solution (find c)

$y = (3e)(e^{-\frac{1}{x}})$