Math

January 16th

Part I

Si loomis
Sears 540

Trig intiglals

Hw questions

Mpod 2 Question 7

$$\int \ln(1+x^2)\,dx$$

$u = \ln(1+x^2)$ $\quad dv = dx$ $\quad \rightarrow \quad x\ln(1+x^2) - \int \frac{2x^2}{1+x^2}\,dx$

$du = \frac{2x}{1+x^2}$ $\quad v = x$

bring 2 out

$$= 2\int 1 - \frac{1}{1+x^2} \leftarrow 2\int \frac{x^2+1-1}{1+x^2}$$

$$= 2(x - \arctan x)$$

$$x\ln(1+x^2) - 2(x - \arctan x + C)$$

Moving on to Trig intiglals (1)

→ things to wolly abt

3 cases to deal with

(A) $\int \sin^M x \cos^N x\,dx$

(B) $\int \tan^M x \sec^N x\,dx$

(C) $\int \cot^M x \csc^N x\,dx$

(1) if N is odd $u = \sin x$!

Exp $\int \sin^4 x \cos^{3\,(\text{odd})} x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$dx = \frac{du}{\cos x}$

$$\int u^4 \cos^3 x \frac{du}{\cos x}$$

$$\int u^4 (1-u^2)\,du$$

$$= \int u^4 - u^6\,du$$

$$= \frac{u^5}{5} - \frac{u^7}{7} = \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + C$$

(2) M is odd $u = \cos x$!

Exp $\int \cos^5 x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$$\int \cos^5 x \frac{du}{\cos x}$$

$$\int (\cos^2 x)^2 \rightarrow (1-\sin^2 x)^2$$

$$\int (1-u^2)^2\,du$$

$$\int 1 - 2u^2 + u^4$$

$$u - \frac{2u^3}{3} + \frac{u^5}{5} = \sin x - \frac{2\sin^3 x}{3} + \frac{\sin^5 x}{5} + C$$

Exp $\int \sin^3 x \cos^5 x\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$dx = \frac{du}{-\sin x}$

$$\int \sin^3 x\, u^5 \frac{du}{-\sin x} = \int (1-u^2)u^5\,dx$$

$$= -\int u^5 - u^7\,du$$

$$= -\left(\frac{u^6}{6} - \frac{u^8}{8}\right) = \frac{\cos^8 x}{8} - \frac{\cos^6 x}{6} + C$$

Maths

Example $\int \sin x^{57} \cos^3 x \, dx$ BOTH ODD

the bigger one → $u = \sin x$

$dx = \frac{du}{\cos x}$

$\int u^{57} \cos^3 x \frac{du}{\cos x} = \int u^{57}(1-u^2)\,du$

$= \int \frac{u^{58}}{58} - \frac{u^{60}}{60} = \boxed{\frac{\sin^{58} x}{58} - \frac{\sin^{60} x}{60} + C}$

③ BOTH EVEN use two trig identities

1# $\sin^2 x = \frac{1-\cos 2x}{2}$ $\cos^2 x = \frac{1+\cos 2x}{2}$

Example $\int \sin^2 x \cos^2 x \, dx = \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) dx$

$= \frac{1}{4}(1-\cos^2 2x)\,dx = \frac{1}{4}\int (\sin^2 2x)\,dx = \int \frac{1-\cos 4x}{2}\,dx$

$= \frac{1}{8}\left[x - \frac{\sin 4x}{4}\right] + C$

⑧ $\int \tan^M x \sec^N x \, dx$

① N is even $u = \tan x$

Exp: $\int \tan^3 x \sec^4 x \, dx$

$u = \tan x$ $du = \sec^2 x \, dx$

$dx = \frac{du}{\sec^2 x}$

$= \int u^3 \sec^4 x \frac{du}{\sec^2 x}$

$= \int u^3 (u^2+1)$

$\frac{u^6}{6} + \frac{u^4}{4}$

$\boxed{\frac{\tan^6 x}{6} + \frac{\tan^4 x}{4} + C}$

Identities

$(\sin^2 x + \cos^2 x = 1)$

$(\tan^2 x + 1 = \sec^2)$

math

(2) M is ODD

$u = \sec x$

Exp $\int \tan^3 x \sec^4 x \, dx$

$u = \sec$

$du = \sec x \tan x \, dx$

$dx = \frac{du}{\sec x \tan x}$

$= \int \tan^3 x \, u^4 \frac{du}{\sec x \tan x}$

$= \int \tan^2 x \, u^3$

$\tan^2 = \sec^2 - 1$

$(u^2 - 1) u^3 \, du$

$u^5 - u^3 \, du$

$\frac{u^6}{6} - \frac{u^4}{4} = \frac{\sec^6 x}{6} - \frac{\sec^4 x}{4} + C$

Evil

(3) M Even N odd

RUN AWAY

?! !

(C) $\int \cot^M x \csc^N x \, dx$

#1 N Even $u = \cot$

#2 M ODD $u = \csc$

#3 M eEen N ODD

RUN