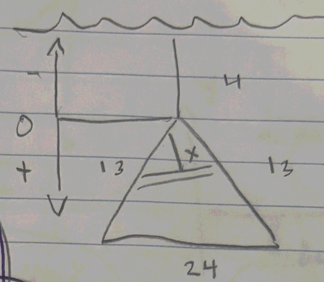
pt. 1

MAth Center of mass

HW Questions:

MPod #4



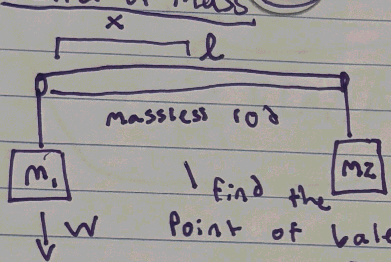
$$F = \int_a^b \rho g\, h(x)\, w(x)\, dx$$

$h(x) = 4 + x$

$\frac{x}{w} = \frac{24}{5}$ $\quad w = \frac{24x}{5}$

$$\int_0^5 (62.4)(4+x)\left(\frac{24x}{5}\right) dx$$

Center of mass



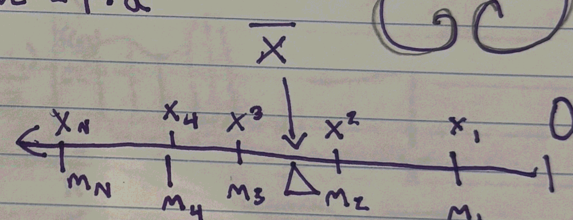
find the point of balence

τ = torque = f · d

$\tau_{m_1} = \tau_{m_2}$

↓

$m_1 g x = m_2 g (l - x)$



$m_n g(x_n - \bar{x}) + (x_4 - \bar{x}) m_4 g = m_1 g(\bar{x} - x_1) + m_2 g(\bar{x} - x_2) + m_3 g(\bar{x} - x_3)$

center doesnt matter in

$m_1 g(\bar{x} - x_1) + m_2 g(\bar{x} - x_2) + m_3 g(\bar{x} - x_3) + m_4 g(\bar{x} - 4) \dots m_n g(\bar{x} -$

bring the g out - cancle it

$m_1\bar{x} + m_2\bar{x} + m_3\bar{x} + m_4\bar{x} \dots m_N\bar{x} = m_1 x_1 + m_2 x_2 \dots m_n x$

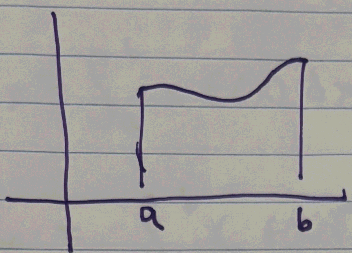
factor

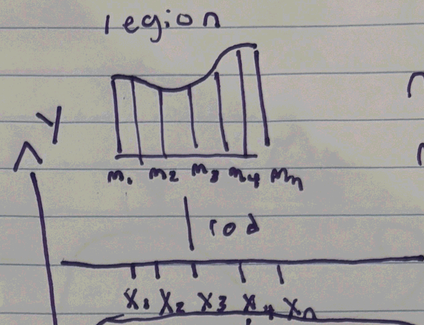
$$\bar{x} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 \dots + m_N x_n}{m_1 + m_2 + m_3 + \dots m_N} = \frac{\sum_{i=1}^{N} x_i m_n}{\sum_{i=1}^{N} m_i}$$

pt. 2

Math

Center of mass





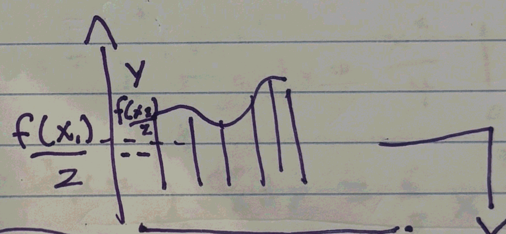
$m_1 = f(x_1)\Delta x$

$m_2 = f(x_2)\Delta x$

ect...

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i f(x_i)\Delta x}{\sum_{i=1}^{N} f(x_i)\Delta x} \leftarrow \bar{x} = \frac{\sum_{i=1}^{N} x_i m_i}{\sum_{i=1}^{N} m_i}$$

$$\bar{x} = \frac{\int_a^b x f(x)\,dx}{\int_a^b f(x)\,dx}$$



$$\bar{y} = \frac{\sum_{i=1}^{N} \frac{f(x_i)}{2} f(x_i)\,dx}{\sum_{i=1}^{N} f(x_i)\Delta x}$$

$$\bar{x} = \frac{\int_a^b x f(x)\,dx}{\int_a^b f(x)\,dx}$$

$$\bar{y} = \frac{\int_a^b \frac{f^2(x)}{2}\,dx}{\int_a^b f(x)\,dx}$$

pt.3

Mathe

center of mass

$M_y = \int_a^b x\,f(x)\,dx$ $\qquad \bar{X} = \frac{M_y}{M}$

$M_x = \int_a^b \frac{(f(x))^2}{2}$ $\qquad \bar{Y} = \frac{M_x}{M}$

$M = \int_a^b f(x)\,dx$

EXP1. $\quad y = \sqrt{x} \quad [0,4]$

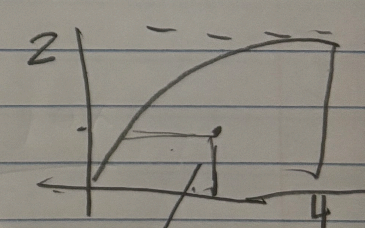
$M_y = \int_a^b x\,f(x)\,dx = \int_0^4 x\sqrt{x}\,dx = \frac{2}{5}x^{\frac{5}{2}}\Big|_0^4 = \frac{64}{5}$

$M_x = \int_a^b \frac{f(x)^2}{2}\,dx = \int_0^4 \frac{(\sqrt{x})^2}{2}\,dx = \frac{x^2}{4}\Big|_0^4 = 4$

$M = \int_a^b f(x)\,dx = \int_0^4 \sqrt{x}\,dx = \frac{2}{3}x^{\frac{3}{2}}\Big|_0^4 = \frac{16}{3}$

$\bar{X} = \frac{M_y}{M} = \frac{\left(\frac{64}{5}\right)}{\left(\frac{16}{3}\right)} = \frac{\cancel{64}}{5}\cdot\frac{3}{\cancel{16}} = \frac{12}{5}$

$\bar{Y} = \frac{M_y}{M} = \frac{4}{\left(\frac{16}{3}\right)} = \frac{12}{16} = \frac{3}{4}$



$\left(\frac{12}{5}, \frac{3}{4}\right)$

Pt. 4

MAth

you can get 0 4 ways
- u sub
- trig sub
- odd func / symetric

Exp 2 $f(x) = \sqrt{4-x^2}$ $[-2,2]$

$$m_y = \int_a^b x f(x)\,dx = \int_{-2}^{2} x\sqrt{4-x^2}\,dx = 0$$

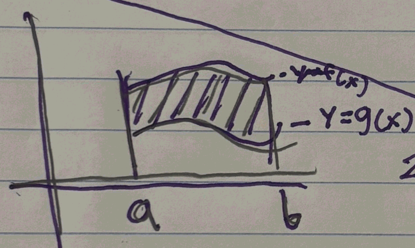
$$m_x = \int_a^b \frac{(f(x))^2}{2}\,dx = \int_{-2}^{2} \frac{(\sqrt{4-x^2})^2}{2}\,dx = \frac{1}{2}\int_{-2}^{2}(4-x^2)\,dx = \frac{16}{3}$$

$$m = \int_a^b f(x)\,dx = \int_{-2}^{2}\sqrt{4-x^2}\,dx = 2\pi$$

$$\bar{X} = \frac{m_y}{m} = \frac{0}{2\pi} = 0$$

$$\bar{Y} = \frac{M_x}{m} = \frac{\left(\frac{16}{3}\right)}{2\pi} = \frac{8}{3\pi}$$

$$\left(0, \frac{8}{3\pi}\right)$$



2 functions

$$m_1 = (f(x_1) - g(x_1))\Delta x$$

$$m_2 = (f(x_2) - g(x_2))\Delta x$$

$$m_y = \int_a^b x(f(x) - g(x))\,dx$$

$$m_x = \int_a^b \frac{(f(x))^2 - (g(x))^2}{2}\,dx$$

$$m = \int_a^b (f(x) - g(x))\,dx$$