MATH: January 14

➪ Intigration by Parts

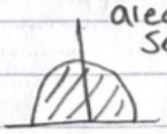
➪ Quiz 1 overview

① $\int (3x^2+2x+1)\,dx = x^3+x^2+x+C$

② $\int (\sin x + \frac{2}{x} + \frac{3}{e^x})\,dx = \int (\sin x + 2x^{-1} + 3e^{-x})\,dx$

$= -\cos x + 2\ln|x| - 3e^{-x} + C$

③ $\int_{-1}^{1} \sqrt{1-x^2}\,dx =$ area of semi-circle



④ $\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx$

$u = 1+\sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$dx = 2\sqrt{x}\,du$

$\int \frac{2\sqrt{x}\,du}{\sqrt{x}(u)^2} = \frac{-2}{u} = \frac{-2}{1+\sqrt{x}} + C$

⑤ $A + AB + ABC + ABCD = 1395$

$ABCD$
ABC
AB
A
1395

→

$AAAA$
BBB
CC
D
1395

$A = 1$

BBB
CC
D
284

$B = 2$

$1395 - 1111$

CC
D
62

$C = 5$

$D = 7$

MATH January 14 Part II

intiglation by Parts

Formula

$$(UV)' = U\,dv + V\,dv$$

$$\int U\,dv = \int (UV)' - \int V\,dv$$

called the Parts formula

$$\int U\,dv = UV - \int V\,dv$$

? ? ? ? Chris Butler Statement ?

law: dx always has to go after the dv?

Suggestion: if you have a polinomial make it U if you have an exponential or trig thing make it dv

Examples

① $\int xe^x\,dx$

$U = x \quad dv = e^x dx$

$du = dx \quad v = e^x$

$\rightarrow xe^x - \int e^x dx = xe^x - e^x + C$

② $\int x^2 \cos x\,dx \longrightarrow x^2 \sin x - \int 2x \sin x\,dx$

$U = x^2 \quad dv = \cos dx$

$du = 2x dx \quad V = \sin(x)$

hit it again

$U = 2x \quad dv = \sin x dx$

$du = 2dx \quad V = -\cos x$

$x^2 \sin x - (-2x \cos x - \int -2\cos x dx$

$\rightarrow x^2 \sin x + 2x \cos x - 2\sin x + C$

③ $\int x \ln x\,dx$

$U = \ln x \quad dv = x\,dx$

$du = \frac{1}{x} \quad v = \frac{x^2}{2}$

Chris's dont screw yourself rule: dont put anything v can't intiglate equal to dv

$\frac{x^2}{2}\ln x - \int \frac{x^2}{2}\frac{1}{x} \rightarrow \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$

steps
- choose a U, choose a dv
- du = derivitive of U
- v = intigral of dv

Part III

MATH

January 14

Toy's R US Rule
everyone has to get something

④ $\int \ln x \, dx$

$u = \ln x \quad dv = 1\,dx$

$du = \frac{1}{x}dx \quad v = x$

$\rightarrow x\ln x - \int x \cdot \frac{1}{x}dx \rightarrow \boxed{x\ln x - x + C}$

⑤ $\int \text{Arctan}\, x \, dx$ ← give it the number 1

$u = \text{Arctan} \quad dv = 1\,dx$

$du = \frac{1}{1+x^2}dx \quad v = x$

$\rightarrow x\,\text{Arctan}\, x - \int \frac{x}{1+x^2}dx$

$u = 1+x^2$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$\rightarrow \boxed{x\,\text{Arctan}\, x - \frac{1}{2}\ln|1+x^2| + C}$

⑥ $\int e^{\sqrt{x}}\,dx$?

$k = \sqrt{x} \quad dx = 2\sqrt{x}\,dk$

$dk = \frac{1}{2\sqrt{x}}$

$= \int e^k 2\sqrt{x}\,dk = \int 2k e^k\,dk$

$u = 2k \quad dv = e^k dk \rightarrow 2ke^k - \int 2e^k dk$

$du = 2\,dk \quad v = e^k$

$\rightarrow 2ke^k - 2e^k$

$= \boxed{2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C}$

⑦a $\int x\sqrt{x-1}\,dx \quad u = x-1 \quad x = u+1$

$du = dx$

$\int (u+1)\sqrt{u}\,du \rightarrow \int u^{\frac{3}{2}} + u^{\frac{1}{2}})\,du \rightarrow \frac{2u^{\frac{5}{2}}}{5} + \frac{2}{3}u^{\frac{3}{2}}$

$= \boxed{\frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + C}$

$\rightarrow x\left(\frac{2(x-1)^{\frac{3}{2}}}{3}\right) - \int \frac{2}{3}(x-1)^{\frac{3}{2}}\,dx$

$\left(\frac{2(x-1)^{\frac{2}{3}}}{3}\right) - \frac{2}{5}\left(\frac{2(x-1)^{\frac{5}{2}}}{5}\right)$

⑦b $\int x\sqrt{x-1}\,dx$

$u = x \quad dv = \sqrt{x-1}\,dx$

$du = dx \quad v = \frac{2(x-1)^{\frac{3}{2}}}{3}$

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$$\frac{2x(x-1)^{3/2}}{3} - \frac{4}{15}(x-1)^{5/2} + C$$

di: