

# Math: Feb 11<sup>th</sup>

## fluid force

quiz recap

①  $P(x) = kx^2 e^{-x^3} \quad x \geq 0$

$$\int_0^{\infty} kx^2 e^{-x^3} dx \quad u = -x^3 \quad du = -3x^2 dx \quad \int kx^2 e^u \frac{du}{-3x^2}$$

$$\int -\frac{k}{3} e^u \quad -\frac{k}{3} e^u \Big|_0^{\infty} = \frac{k}{3} = 1 \quad \boxed{k=3}$$

②  $y = \frac{2}{3}(x^2+1)^{\frac{3}{2}} \quad [1, 4]$

$$\frac{dy}{dx} = (x^2+1)^{\frac{1}{2}} (2x)$$

$$\left(\frac{dy}{dx}\right)^2 = (x^2+1)(4x^2) \rightarrow 4x^2 + 4x^4$$

$$S = \int_1^4 \sqrt{(2x^2+1)^2} dx \rightarrow \int_1^4 2x^2+1 dx$$

$$= \frac{2x^3}{3} + x \Big|_1^4 = \boxed{45}$$

③  $y = \frac{x^3}{3} \quad [0, 1]$

$$y' = x^2$$

$$y'^2 = x^4$$

$$1+y'^2 = 1+x^4$$

$$SA = \int_0^1 2\pi \left(\frac{x^3}{3}\right) \sqrt{1+x^4}$$

$$u = 1+x^4, \quad du = 4x^3 dx, \quad dx = \frac{du}{4x^3}$$

$$\frac{2\pi}{3} \int x^3 \sqrt{u} \frac{du}{4x^3}$$

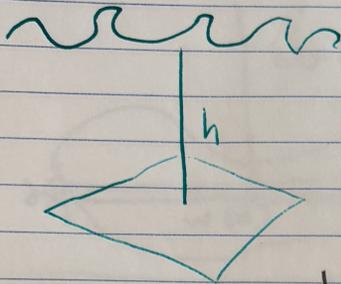
$$\frac{1}{4} \left[ \frac{2\pi}{3} \left( \frac{2}{3} u^{\frac{3}{2}} \right) \Big|_0^1 \right]$$

$$\boxed{\frac{\pi}{9}(2^{\frac{3}{2}} - 1)}$$

# MATH

Feb 11  
fluid force

fluid pressure / fluid force



Pressure  $P = \rho g h$

$\rho$  = density  
 $g$  = gravity  
 $h$  = depth

mass density  $\rho_{\text{water}} = 10^3 \text{ kg/m}^3$   $g = 9.81 \text{ m/s}^2$

force density  $\rho g = 9810 \text{ N/m}^3 = 62.4 \text{ lb/ft}^3$

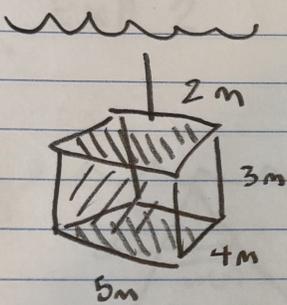
Sometimes just written

$\rho = 9810 \text{ N/m}^3$

Force = Pressure  $\cdot$  Area

$F = P \cdot A$

Exp 1



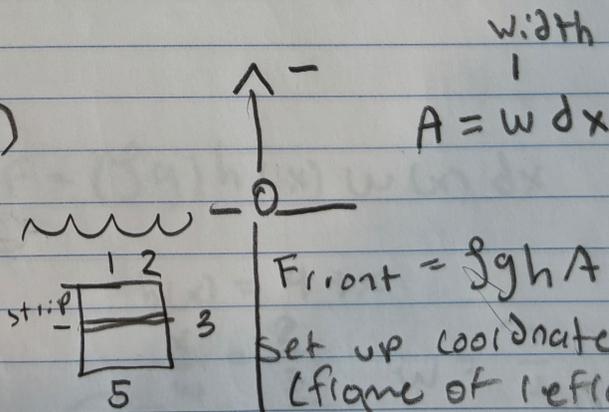
what is the force on the top?

$F_T = P \cdot A$

$F_T = \rho g h \cdot A$

$F_T = 9810 (2) (4 \cdot 5)$

$F_B = PA = \rho g h A$   
 $= 9810 (5) (5 \cdot 4)$



Front =  $\rho g h A = (9810) x$   
set up coordinate system  
(frame of reference)

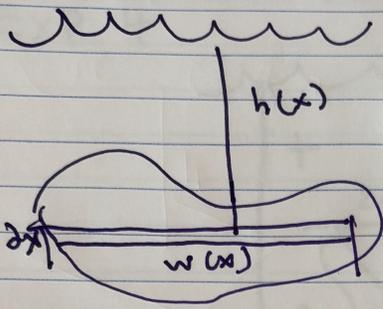
$F_{\text{front}}$  - not a constant depth

$F_{\text{front}} = \int_2^5 (9810) x 5 dx$

# MATH

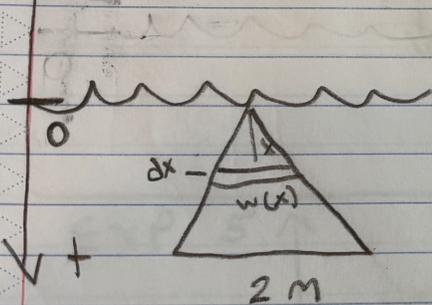
Feb 11th

fluid force



$$F = \int_a^b \rho g h(x) w(x) dx$$

## EXP 2



$$F = \int_a^b (\rho g) h(x) w(x) dx$$

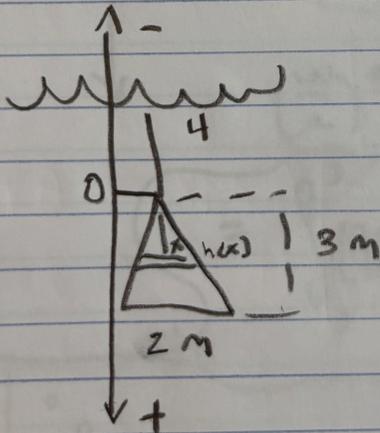
$$= \int_0^3 9810 x \left(\frac{2}{3}x\right) dx$$

$$h(x) = x$$

$$\frac{x}{w} = \frac{3}{2}$$

$$w = \frac{2}{3}x$$

## EXP 3



$$F = (\rho g) h(x) w(x) dx$$

$$h(x) = 4 + x$$

$$\frac{x}{w} = \frac{2}{3} \quad w = \frac{2x}{3}$$

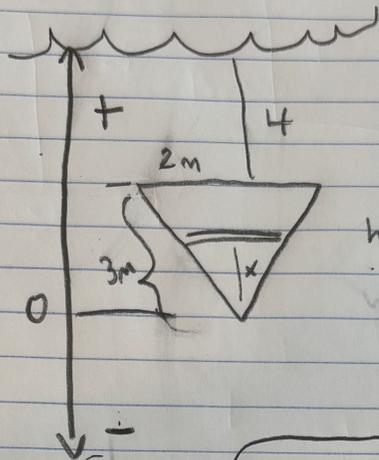
$$\int_0^3 9810 (4+x) \left(\frac{2x}{3}\right) dx$$

# MATH

Feb 11<sup>th</sup>

## fluid Force

fluid force / fluid Pressure  
EXP 4



$$F = \int_a^b \rho g h(x) w(x) dx$$

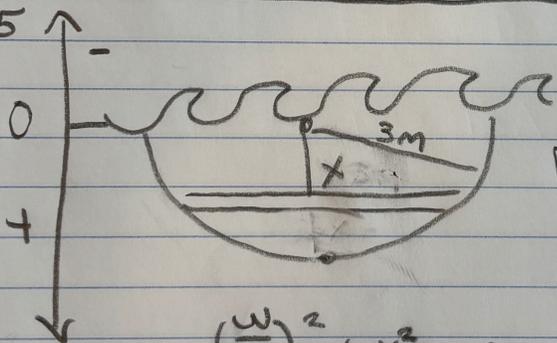
$$h(x) = (7-x)$$

$$\frac{x}{w} = \frac{3}{2}$$

$$w = \frac{2}{3}x$$

$$F = \int_0^3 9810 (7-x) \left(\frac{2x}{3}\right) dx$$

EXP 5



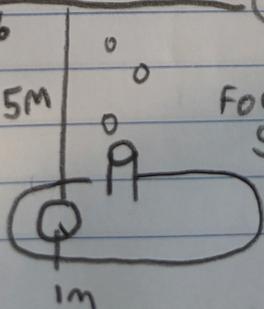
$$F = \int_a^b \rho g h(x) w(x) dx$$

$$h(x) = x$$

$$\left(\frac{w}{2}\right)^2 + x^2 = 9 \quad w = 2\sqrt{9-x^2}$$

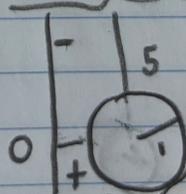
$$= \int_0^3 (9810) (x) 2\sqrt{9-x^2} dx$$

EXP 6



Force on the sub viewing window  $\rightarrow$

$$F = \int_a^b \rho g h(x) w(x) dx$$



$$h(x) = 6+x$$

$$w = 2\sqrt{1-x^2}$$

$$= \int_{-1}^1 9810 (6+x) (2\sqrt{1-x^2}) dx$$