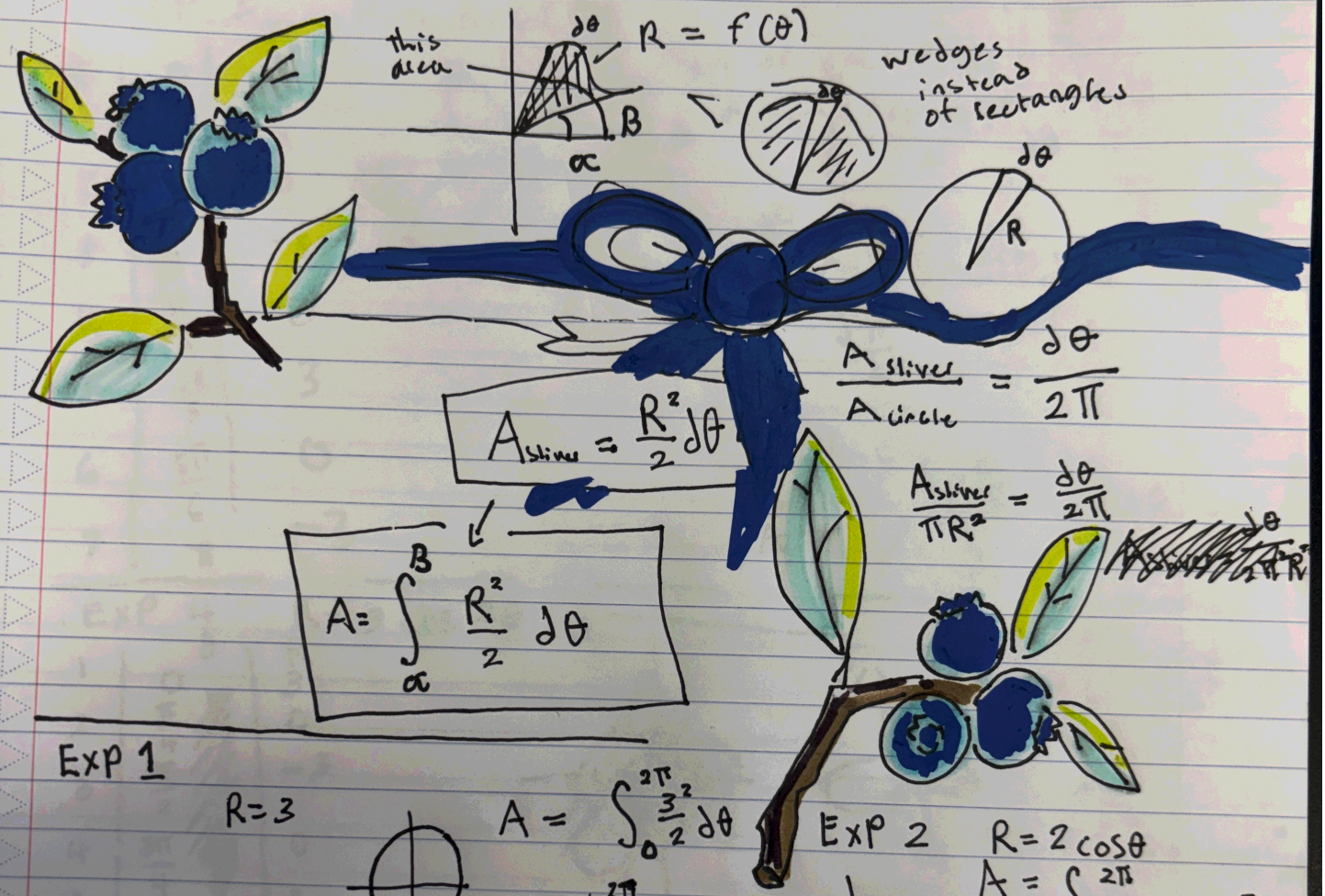
API: 1 10th

MATH Polar Area



$$\frac{A_{sliver}}{A_{circle}} = \frac{d\theta}{2\pi}$$

$$A_{sliver} = \frac{R^2}{2} d\theta$$

$$\frac{A_{sliver}}{\pi R^2} = \frac{d\theta}{2\pi}$$

$$A = \int_{\alpha}^{B} \frac{R^2}{2} d\theta$$

Exp 1

$R = 3$

$$A = \int_0^{2\pi} \frac{3^2}{2} d\theta$$

$$\frac{9}{2\theta} \Big|_0^{2\pi} = 9\pi$$

Exp 2 $R = 2\cos\theta$

$$A = \int_0^{2\pi} \frac{(2\cos\theta)^2}{2} d\theta$$

$$= 2\int_0^{2\pi} \cos^2\theta \, d\theta$$

$$= \int_0^{\pi} 1 + \cos 2\theta \, d\theta$$

$$= 2\pi$$

but $R = 2\cos\theta \rightarrow$
$(x-1)^2 + y^2 = 1$
circle radius 1 Area π
but $2\cos\theta$ goes around the circle twice

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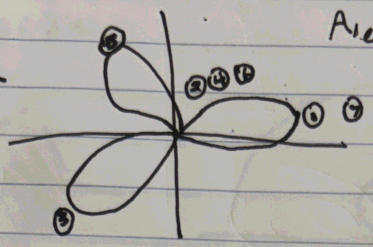
Polar Area

Exp 3 $R = 3\cos 3\theta$

#	θ	R
1	0	3
2	$\frac{\pi}{6}$	0
3	$\frac{\pi}{3}$	-3
4	$\frac{\pi}{2}$	0
5	$\frac{2\pi}{3}$	3
6	$\frac{5\pi}{6}$	0
7	π	-3



Area of one Petal

$$A = \frac{1}{3}\int_0^{\pi} \frac{(3\cos 3\theta)^2}{2}\, d\theta$$

or

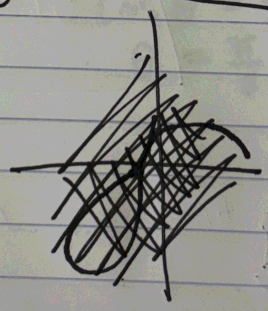
$$A = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{(3\cos 3\theta)^2}{2}\, d\theta$$

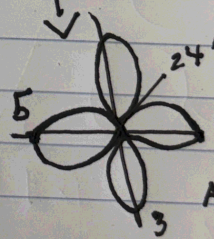
or

$$A = 2\int_0^{\frac{\pi}{6}} \frac{(3\cos\theta)^2}{2}\, d\theta$$

Exp 4 $R = 3\cos 2\theta$

	θ	R
1	0	3
2	$\frac{\pi}{4}$	0
3	$\frac{\pi}{2}$	-3
4	$\frac{3\pi}{4}$	0
5	π	3





$$A = \frac{1}{4}\int_0^{2\pi} \frac{(3\cos 2\theta)^2}{2}\, d\theta$$

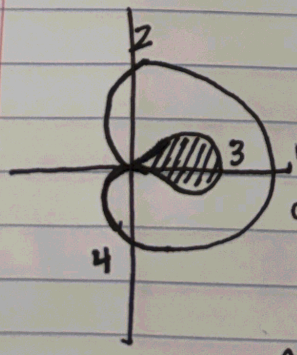
or

$$A = 2\int_0^{\frac{\pi}{4}} \frac{(3\cos 2\theta)^2}{2}\, d\theta$$

Exp 5 Area of inner loop

$R = 1 + 2\cos\theta$

$$A = \int \frac{(1+2\cos\theta)^2}{2}\, d\theta$$



need a point $2 < x < 3$

and $R = 0$ $\quad 1 + 2\cos\theta = 0$

$\theta = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$ ← not in the range

	θ	R
1	0	3
2	$\frac{\pi}{2}$	1
3	π	-1
4	$\frac{3\pi}{2}$	1
5	2π	3

$\frac{2\pi}{3}$

$$A = 2\int_{\frac{2\pi}{3}}^{\pi} \frac{(1+2\cos\theta)^2}{2}$$

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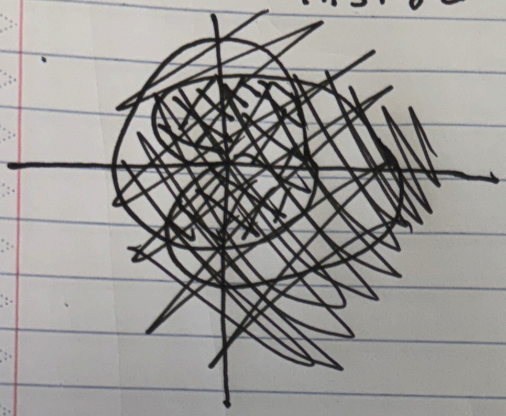
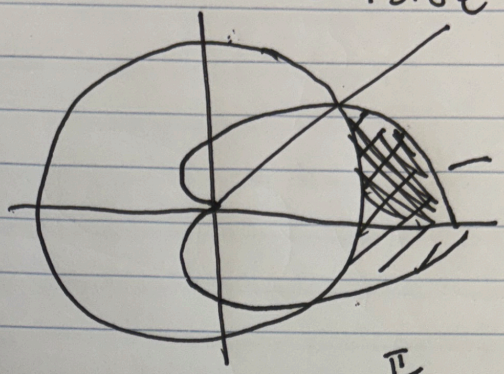
Polar Area

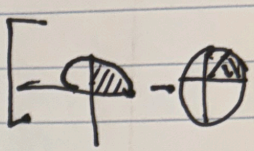
EXP 6 $R = 4 + 4\cos\theta$ $R = 6$

inside



outside



$= 2[$  $]$

$$2\left[\int_0^{\frac{\pi}{3}} \frac{(4+4(\cos\theta))^2}{2}\,d\theta - \int_0^{\frac{\pi}{3}} \frac{6^2}{2}\,d\theta\right]$$

$$4 + 4\cos\theta = 6$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$