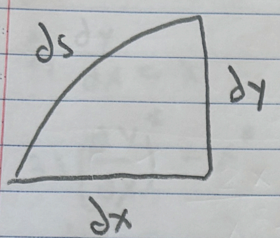
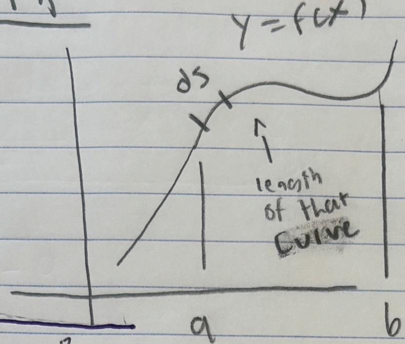
MATH feb 9

Arc length ect
and surface area

Arclength





- this is a calc class

Chop it up into little pieces

$$\sqrt{ds^2 = \frac{dx^2 + dy^2}{dx^2} dx^2}$$

$$S = \int_a^b ds = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Arc length Formula

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Exp 1 $y = x$ $[0, 2]$

$y = x$

$\frac{dy}{dx} = 1$

$\left(\frac{dy}{dx}\right)^2 = 1$

$1 + \left(\frac{dy}{dx}\right)^2 = 2$

$S = \int_0^2 \sqrt{2}\, dx = \sqrt{2}x \Big|_0^2 = 2\sqrt{2}$

Exp 3 $y = x^2 - \frac{\ln x}{8}$ $[1, 3]$

1.) $y = x^2 - \frac{\ln x}{8}$

2.) $\frac{dy}{dx} = 2x - \frac{1}{8x}$

3.) $\left(\frac{dy}{dx}\right)^2 = 4x^2 - \frac{1}{2} + \frac{1}{64x^2}$

4.) $1 + \left(\frac{dy}{dx}\right)^2 = 1 + 4x^2 - \frac{1}{2} + \frac{1}{64x^2}$

$S = \int_1^3 \sqrt{\left(2x + \frac{1}{8x}\right)^2}\, dx$

$= \int_1^3 \left(2x + \frac{1}{8x}\right) dx$

$= 8 + \frac{\ln 3}{8}$

Exp 2 $y = x^{3/2}$ $[1, 4]$

$y = x^{\frac{3}{2}}$

$\frac{dy}{dx} = \frac{3}{2} x^{\frac{1}{2}}$

$\frac{dy}{dx}^2 = \frac{9}{4} x$

$+1 = 1 + \frac{9x}{4}$

$S = \int_1^4 1 + \frac{9x}{4}\, dx$

$=$ U-Sub

$= \frac{8}{27}\left(1 + \frac{9}{4}x\right)^{2/3} \Big|_1^4$

$= \frac{8}{27}\left(10^{\frac{3}{2}} - \frac{13}{4}^{3/2}\right)$

Math

feb 9th

Arc length ect

Exp 4 $y = \frac{x^2}{2}$ $[0, 1]$

1.) $y = \frac{x^2}{2}$

2.) $\frac{dy}{dx} = x$

3.) $\left(\frac{dy}{dx}\right)^2 = x^2$

$+1$

4.) $= x + 1$

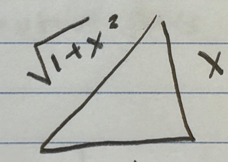
5.) $S = \int_0^1 \sqrt{1 + x^2}\, dx$

trig sub $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$= \int \sqrt{1 + \tan^2\theta}\, \sec^2\theta\, d\theta$

$= \int \sec^3\theta\, d\theta$

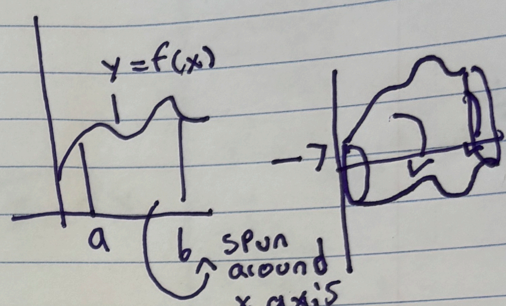


$= \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta|$

$= \frac{1}{2}\sqrt{1+x^2}\, x + \frac{1}{2}\ln|\sqrt{1+x^2} + x| \Big|_0^1$

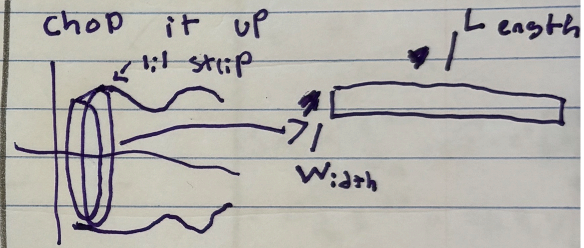
$=$

Surface area



don't worry about the circles on the end

chop it up

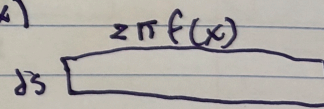


Length = Circumfrance = $2\pi r$

$r = f(x)$

$L = 2\pi f(x)$

$w = ds$



Area = $2\pi f(x)\, ds$

$= 2\pi f(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

intigiate it

Surface Area Formula =

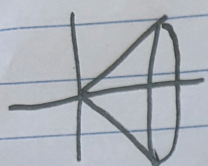
$$SA = \int_a^b 2\pi f(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Math (feb 9th Mr. Lara)

arc length ect

surface area examples

Exp 1 $y = x$ $[0, 2]$



$$SA = \int_0^2 2\pi x \sqrt{2}\, dx$$

$$= 4\sqrt{2}\,\pi$$

$y = x$

$\frac{dy}{dx} = 1$

$\left(\frac{dy}{dx}\right)^2 = 1$

$1 + \left(\frac{dy}{dx}\right)^2 = 2$

Exp 2 about x-axis

$y = x^3$ $[0, 1]$

$\frac{dy}{dx} = 3x^2$

$$SA = \int_0^1 2\pi x^3 \sqrt{9x^4+1}\, dx$$

$\left(\frac{dy}{dx}\right)^2 = 9x^4$

$+1$

$= 9x^4 + 1$

$u = 1 + 9x^4$

$du = 36x^3\, dx$

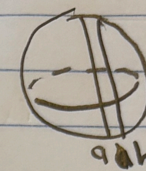
$$= 2\pi \int_0^1 x^3 \sqrt{u}\, \frac{du}{36x^3}$$

$$= 2\pi \int \sqrt{u}\, du\, \frac{1}{36} = \frac{\pi}{18} \frac{2}{3} u^{3/2} = \frac{\pi}{27}(1 + 9x^4)^{3/2}\Big|_0^1$$

$$= \frac{\pi}{27}\left(10^{3/2} - 1\right)$$

Exp 3 $y = \sqrt{R^2 - x^2}$

$[a, a+h]$



$y' = \frac{1}{2}(R^2 - x^2)^{-\frac{1}{2}}(-2x)$

$\frac{-x}{\sqrt{R^2 - x^2}}$

$y'^2 = \frac{x^2}{R^2 - x^2}$

$1 + y'^2 = 1 + \frac{x^2}{R^2 - x^2}$

$= \frac{R^2}{R^2 - x^2}$

$SA = \int_a^{a+h} 2\pi \sqrt{R^2 - x^2} \sqrt{\frac{R^2}{R^2 - x^2}}\, dx$

$= 2\pi \int_a^{a+h} R\, dx = 2\pi R y \Big|_a^{a+h}$

$= 2\pi R(h)$

$2\pi R h$