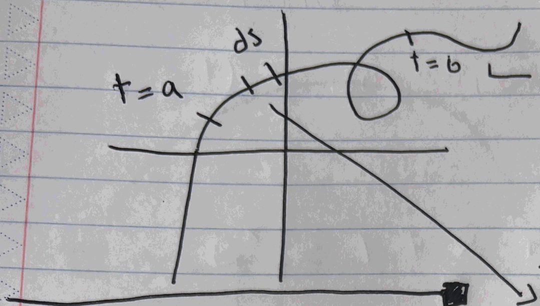
April 6th

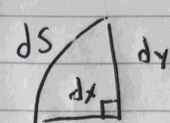
Pt. 1

MATH Palametric

Arc Length:



$c(t) = (x(t), y(t))$



① $\sqrt{ds^2 = \frac{dx^2 + dy^2}{dt^2} dt^2}$

② $dS = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Arcl Length

③ $S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Exp 1

$c(t) = (1+2t, 2+4t)$

$1 \le t \le 4$

$x = 1 + 2t$

$\frac{dx}{dt} = 2$

$\left(\frac{dx}{dt}\right)^2 = 4$

$y = 2 + 4t$

$\frac{dy}{dt} = 4$

$\left(\frac{dy}{dt}\right)^2 = 16$

$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 20$

$\boxed{S = \int_1^4 \sqrt{20}\, dt = 3\sqrt{20}}$

Exp 2 $c(t) = \left(\frac{t^2}{2}, \frac{t^3}{3}\right)$ $0 \le t \le 1$

$x = \frac{t^2}{2}$ $y = \frac{t^3}{3}$

$\frac{dx}{dt} = t$ $\frac{dy}{dt} = t^2$

$\frac{dx^2}{dt} = t^2$ $\frac{dy^2}{dt} = t^4$

$S = \int_0^1 \sqrt{t^2 + t^4}\, dt$

$S = \int_0^1 t\sqrt{1+t^2}\, dt$ $u = 1 + t^2$ $\frac{du}{2t} = dt$

$S = \int_1^2 \sqrt{u}\, du \rightarrow \frac{2u^{3/2}}{3} (2) \Big|_0^1$

$\boxed{S = \frac{1}{3}\left((2)^{3/2} - 1\right)}$

April 6th Pt. 2

MATH Parametric

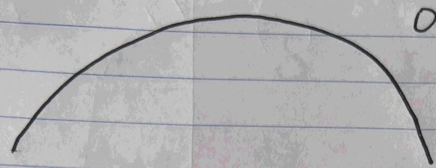
Arc Length

Exp 3

$c(t) = \langle t - \sin t, 1 - \cos t \rangle$

$0 \le t \le 2\pi$

this is called a Hypercycloid



☐ $y = 1 - \cos t$

☐ $\frac{dy}{dt} = \sin t$

☐ $\left(\frac{dy}{dt}\right)^2 = \sin^2 t$

☐ $x = t - \sin t$

☐ $\frac{dx}{dt} = 1 - \cos t$

☐ $\left(\frac{dx}{dt}\right)^2 = (1-\cos t)^2 = 1 - 2\cos(t) + \cos^2 t$

☐ $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 1 - 2\cos(t) + \cos^2 t + \sin^2 t$

$= 2 - 2\cos(t)$

$S = \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$

Identity $\sin^2 t = \frac{1 - \cos 2t}{2}$

$\sin^2\left(\frac{t}{2}\right) = \frac{1 - \cos t}{2}$

$2\sin^2\left(\frac{t}{2}\right) = 1 - \cos t$

$\int_0^{2\pi} \sqrt{\left(2\sin^2\left(\frac{t}{2}\right)\right)\cdot 2} \rightarrow \int_0^{2\pi} \sqrt{2}\sqrt{2}\,\sin\left(\frac{t}{2}\right) \rightarrow \int_0^{2\pi} 2\sin\left(\frac{t}{2}\right)$

$\boxed{S = -2\cos\left(\frac{t}{2}\right)\Big|_0^{2\pi} \rightarrow S = 8}$

April 6th pt. III

MATH Parametric

Arclength

if $S = \int_a^t \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ is distanc

then $\frac{ds}{dt}$ is time $\boxed{\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}}$

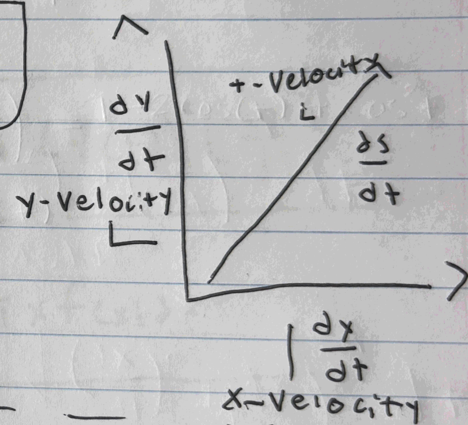
↑ formula for speed

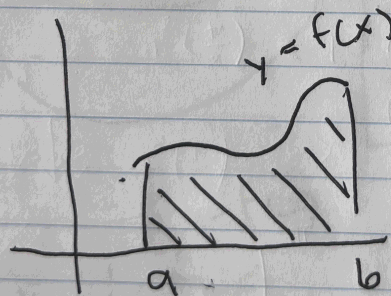
Exp 4 $c(t) = \left(\frac{t^2}{2}, \frac{t^3}{3}\right)$

$\frac{dx}{dt} = t$

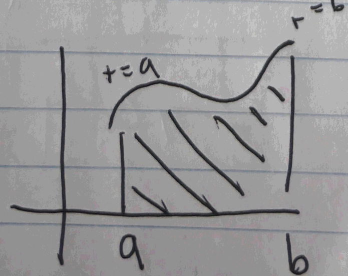
$\frac{dy}{dt} = t^2$

$\boxed{\frac{ds}{dt} = \sqrt{t^2 + t^4}}$





$A = \int_a^b f(x)\, dx$



$c(t) = (x(t), y(t))$

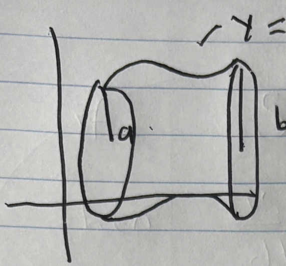
$\frac{dx}{dt} > 0$

$\boxed{A = \int_a^b y(t)\left(\frac{dx}{dt}\right) dt}$

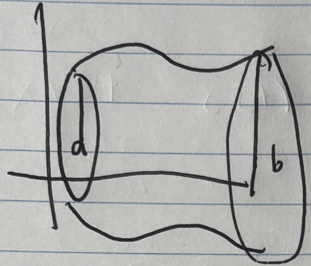
April 6th pt. IIII

MATH

Parametric Arc Length



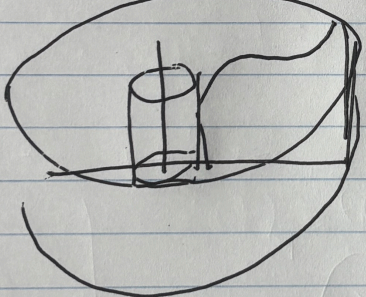
$$V = \int_a^b \pi (f(x))^2 \, dx$$



$$c(t) = (x(t), y(t))$$

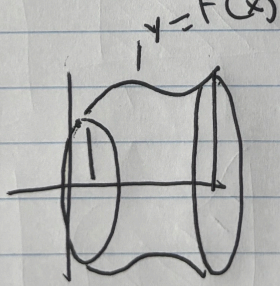
$$\frac{dx}{dt} > 0$$

$$\boxed{V = \int_a^b \pi (y(t))^2 \left(\frac{dx}{dt}\right) dt}$$



$$V = \int_a^b 2\pi x f(x) \, dx$$

$$\boxed{\int_a^b = 2\pi (x(t)) (y(t)) \left(\frac{dx}{dt}\right) dt}$$



$$SA = \int_a^b 2\pi f(x) \overbrace{\sqrt{\left(1 + \frac{dy}{dx}\right)}}^{ds} \, dx$$

$$\boxed{= \int_a^b 2\pi (y(t)) \underbrace{\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}}_{ds} \, dt}$$