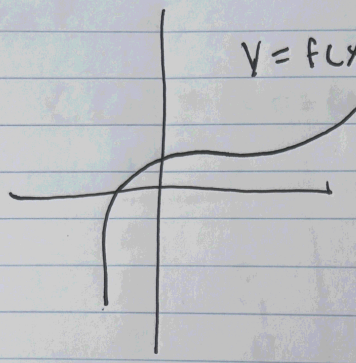
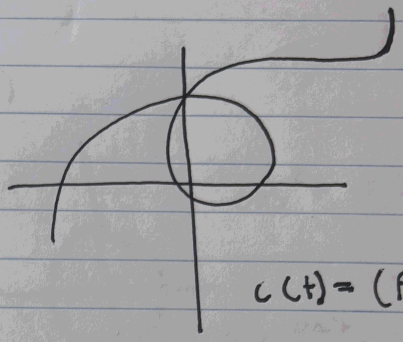
MATH

April 3rd

Parametric equations





no longer passes vertic line test

$c(t) = (f(t), g(t))$

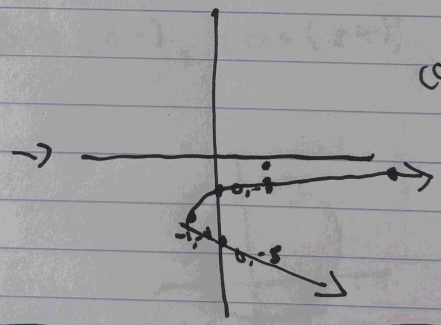
$x = f(t)$

$y = g(t)$

Exp 1

$c(t) = ((t^2 + 2t), (t - 3))$

t	x	y
-2	0	-5
-1	-1	-4
0	0	-3
1	3	-2
2	8	-1



connect the dots

Another way to do it 2nd way

now you dont know which way the function is progressing (increase/decrease)

$x = t^2 + 2t$

$y = t - 3$

$t = y + 3$

$x = (y+3)^2 + 2(y+3)$

3rd way Put calculator in PAR mode

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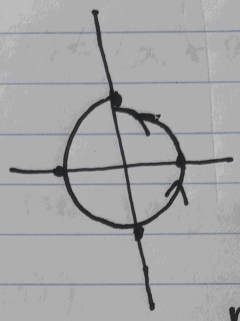
Parametric equations Pt. 2

Exp 2

$C(t) = (\cos(t); \sin(t))$

t	x	y
0	1	0
$\frac{\pi}{2}$	0	1
π	-1	0
$\frac{3\pi}{2}$	0	-1
2π	1	0



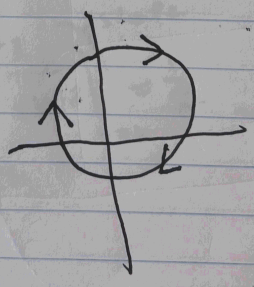
$\cos^2 t + \sin^2 t = 1$

↳ unit circle

need to be in radian mode

Exp 3 $C(t) = (\sin(2t), \cos(2t))$

t	x	y
0	0	1
$\frac{\pi}{4}$	1	0
$\frac{\pi}{2}$	0	-1
$\frac{3\pi}{4}$	1	0
π	0	1



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given a curve, come up with its reparameterization.

$y = f(x) \rightarrow C(t) = (t, f(t))$

Line

(a, b) slope $= m$

$C(t) = (a + Rt, b + St)$

$\frac{S}{R}$ = slope

MATH Parametric equations

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Part 3

How to Parameterize:

Circles

center = (a, b)
Radius = R

$C(t) = (a + R\cos t), (b + R\sin t)$

Finding the slope of the tangent line:

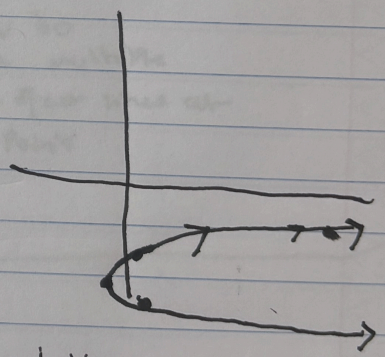
$C(t) = (x(t), y(t))$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} \Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$C(t) = (t^2 + 2t, t - 3)$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \rightarrow \frac{1}{2t+2}\Big|_{t=6} = \frac{1}{2} \quad \text{slope}$$

$$\frac{1}{2t+2}\Big|_{t=-1} = \text{ONE}$$



tangent line

$y + 3 = \frac{1}{2}(x - 0)$

$\downarrow$

$\frac{1}{2t+2}$ = tangent line

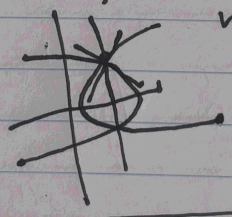
EXP 4 $\quad L(t) = (t^3 - 4t,\ t^2)$

EQ tan line 0,4

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{3t^2-4}\Big|_{t=2} = \frac{4}{8} = \frac{1}{2}$$

$= y - 4 = \frac{1}{2}(x - 0)$ — don't matter

$$\frac{2t}{3t^2-4}\Big|_{t=-2} = \frac{-4}{8} = -\frac{1}{2}$$



So you do have multiple tangent lines at 1 point

Finding Concavity

$L(t) = (x(t), y(t))$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \qquad \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \left(\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}\right)\frac{dt}{dx}$$