Math: Sequences and Series

March 4

we use curly brackets

Sequences

Notation

$\{a_N\}_{N=1}^{\infty}$

this means

$a_1, a_2, a_3 \ldots$

- f.1 • 1, 2, 3, 4, 5, 6, ... ☑ is a sequence
- f.2 • 1, 1, 1, 1, 1, 1 ... ☑
- f.3 • $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6} \ldots$ ☑
- • 1, 1, 2, 3, 5, 8, 13, 21 ... ☑
- • 3, 1, 4, 1, 5, 9, 2, 6 ... ☑
- • 7, 5, 8, 1, 2, 57, 16, 2 ... ☑

All sequences bc they are ordered lists of numbers

Often written as a rule: f.1 $a_N = N$ $\quad \{N\}_{N=1}^{\infty}$

or

f.2 $a_N = 1$ $\quad \{1\}_{N=1}^{\infty}$

f.3 $a_0 = 1 \quad a_1 = 1 \quad a_{N+2} = a_N + a_{N+1}$

3N+1 sequence

$$a_{N+1} = \begin{cases} \frac{a_N}{2} & a_N \text{ is even} \\ 3a_N + 1 & a_N \text{ is odd} \end{cases}$$

Limits

$\lim$ f.1 $= \infty$

$\lim$ f.2 $= 1$

$\lim$ f.3 $= 0$

~~1, 0, 1, 0, 1, 0, 1, 0~~

f.7 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0

limit definition $\{a_N\}_{N=1}^{\infty}$ $\quad \lim_{N \to \infty} a_N = L$

if Given $\varepsilon > 0$ There Exist a N Such that $|a_N - L| < \varepsilon$ For all $n \geq N$

$\lim$ f.7 = DNE

IF $a_N = f(N)$ $\quad \lim_{N \to \infty} f(N) = L$

$\lim_{N \to \infty} a_N = L$

Rules

a $\{a_N\}_{N=1}^{\infty}$ $\quad \{b_N\}_{N=1}^{\infty}$

If:

$\lim_{N \to \infty} a_N = A$ and

$\lim_{N \to \infty} b_N = B$

Then

- ⇒ $\lim_{N \to \infty} c a_N = cA$
- ⇒ $\lim_{N \to \infty} (a_N \pm b_N) = A \pm B$
- ⇒ $\lim_{N \to \infty} a_N b_N = AB$
- ⇒ $\lim_{N \to \infty} \frac{a_N}{b_N} = \frac{A}{B}$

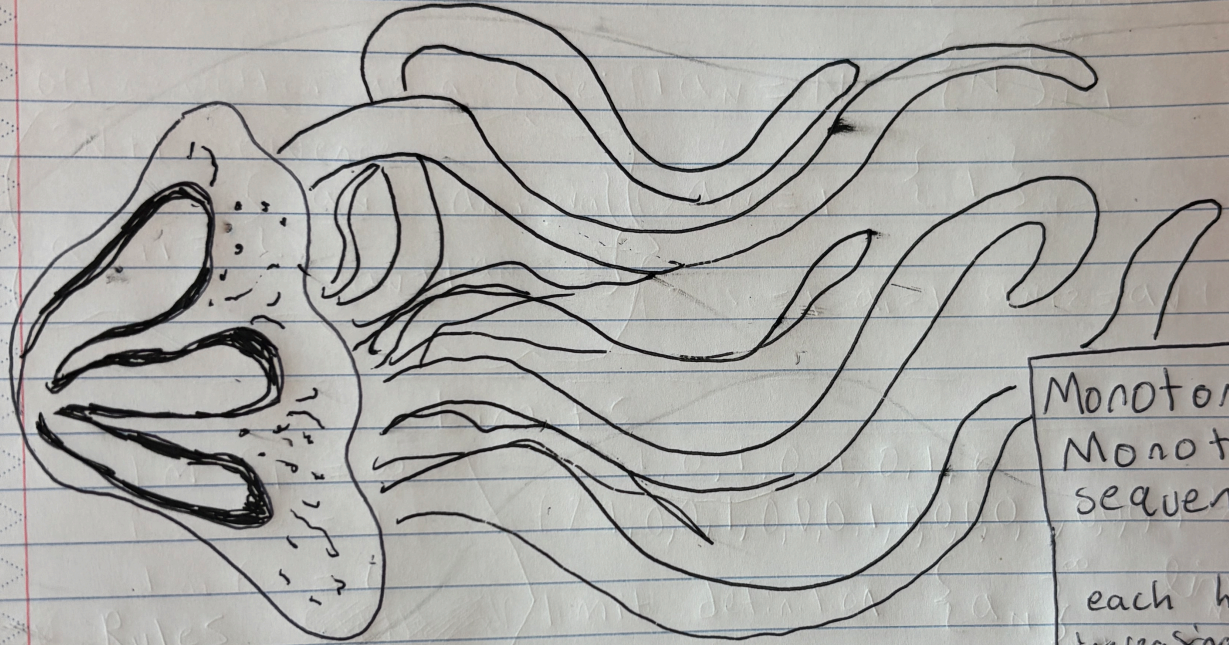
Math

March 4th

you can also use the squeeze thm with sequences

$a_N \le b_N \le c_N$ For all in

$\lim_{N \to \infty} a_N = L$. $\lim_{N \to \infty} c_N = L = \lim_{N \to \infty} b_N = L$



Monotone or Monotonic sequences

each have an

increasing $a_N \le a_{N+1}$

decreasing $a_N \ge a_{N+1}$

valients

you can tell by taking the derivitive

$\lim_{N \to \infty} \frac{\sin N}{N}$: $\frac{-1}{N} \le \frac{\sin N}{N} \le \frac{1}{N}$ ($\frac{-1}{N} \to 0$, $\frac{1}{N} \to 0$)

$\lim_{N \to \infty} \frac{\sin N}{N} = 0$

Bounded

above $a_N \le M$ for all N

Below $a_N \ge m$ for all N

Big THm

INC + Bounded above = Limit

Dec + Bounded Below = Limit