Math

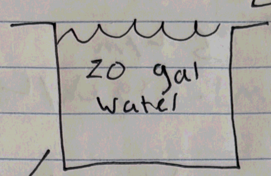
feb 27

aplications of first order linear diferential equations

mixing Problems:

EXP

Put in 4 gal/min water with 2lbs/gal of salt



← Big vat

$y(0) = 0$ starting with pure water

drain at 4 gal per min

How much salt at [redacted]
[redacted] any given time

Salt in − salt out

Let $y =$ # of lbs of salt in vat

(1) $\frac{dy}{dt} = \text{Rate in} - \text{Rate out}$

(2) $\frac{dy}{dt} = \frac{4\,\cancel{gal}}{min} \cdot \frac{2\,lbs}{\cancel{gal}} - \frac{4\,\cancel{gal}}{min} \cdot \frac{y}{20\,\cancel{gal}}$

(3) $\frac{dy}{dt} = 8 - \frac{1}{5}y$

(4) $\frac{dy}{dt} + \frac{1}{5}y = 8$

$P(t) = \frac{1}{5}$

$Q(t) = 8$

$\alpha(t) = e^{\int \frac{1}{5}dt} = e^{\frac{1}{5}t}$

(5) $y = \frac{1}{e^{\frac{1}{5}t}}\left[\int e^{\frac{1}{5}t}(8)\,dt + C\right]$

(6) $y = e^{-\frac{1}{5}t}\left[8 \cdot 5 \cdot e^{\frac{1}{5}t} + C\right]$

(7) $y = e^{-\frac{1}{5}t}\left[40e^{1/5t} + C\right]$ general solution

(8) $y = 40 + Ce^{-\frac{1}{5}t}$

(9) $C = -40$

$$\boxed{y = 40 - 40e^{-\frac{1}{5}t}}$$

MAth

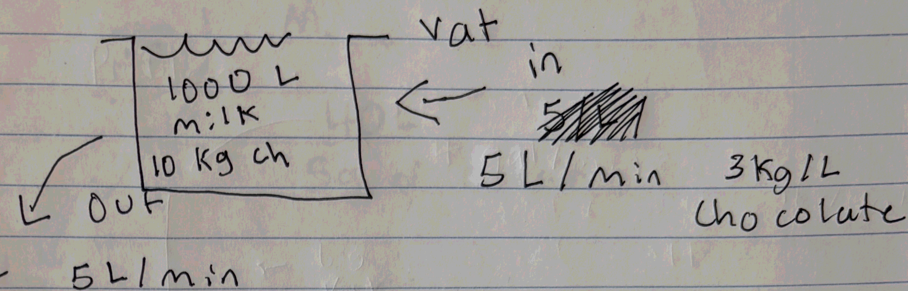
Feb 27th

aplications of first order linear differential equations

Exp 2

ch: chocolate

$Y(0) = 10$



Y = # of kg of ch in vat

① $\frac{dy}{dt}$ = Rate in − Rate out

② $\frac{dy}{dt} = \frac{5L}{min} \cdot \frac{3kg}{L} - \frac{5L}{min} \cdot \frac{Y}{1000}$

③ $\frac{dy}{dt} = 15 - \frac{1}{200} Y$

④ $\frac{dy}{dt} + \frac{1}{200} Y = 15$ $\quad P(t) = \frac{1}{200}$

$Q(t) = 15$

$\mu(t) = e^{\int \frac{1}{200} dt} = e^{\frac{1}{200} t}$

⑤ $Y = e^{-\frac{1}{200} t} \left[\int e^{\frac{1}{200} t} (15)\, dt \right]$

⑥ $y = e^{-\frac{1}{200} t} \left[3000\, e^{\frac{1}{200} t} + C \right]$

⑦ $y = 3000 + C e^{-\frac{1}{200} t}$

⑧ $10 = 3000 + C$

$C = -2990$

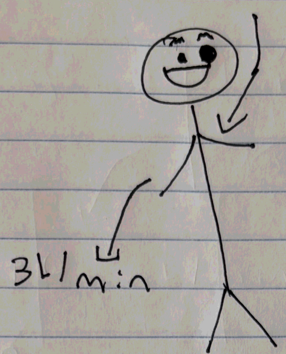
$$\boxed{Y = 3000 - 2990 e^{-\frac{1}{200} t}}$$

MAth

aplications of fisrt older linear equations

Exp 3

2L/m 4g/L cocaine in M.



40L

5g of cocain

Let Y = # of g cocaine in M.

$$\frac{dy}{dt} = \text{Rate in} - \text{Rate out}$$

$$\frac{dy}{dt} = \frac{2L}{min} \cdot \frac{4g}{L} - \frac{3L}{min} \cdot \frac{y}{40-t}$$

$$\frac{dy}{dt} = 8 - \frac{3}{40-t} y$$

$$\frac{dy}{dt} + \frac{3}{40-t} y = 8 \qquad P(t) = \frac{3}{40-t}$$

$$\alpha(t) = e^{\int \frac{3}{40-t} dt} = e^{-3\ln(40-t)} \qquad Q(t) = 8$$

$$\alpha(t) = (40-t)^{-3}$$

$$y = \frac{1}{\alpha(t)} \left[\int \alpha(t) Q(t)\, dt + c \right]$$

$$y = (40-t)^{3} \left[\int (40-t)^{-3} 8\, dt + c \right]$$

$$y = (40-t)^{3} \left[\frac{(40-t)^{-2}}{-2} \cdot 8 + c \right]$$

$$\boxed{y = 4(40-t) + c(40-t)^{3}}$$

$$y(0) = 5$$

$$5 = 4(40-0) + c(40-0)^{3}$$

$$c = \frac{-155}{40^{3}}$$

$$\boxed{y = 4(40-t) - \frac{155}{40^{3}}(40-t)^{3}}$$

$$0 \le t \le 40$$