April 1st

MATH TAYOR Series

~~Quiz Recap~~ april fools im not doing it

Taylor Polynomial formula:

$$f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots \frac{f^N(a)}{N!}(x-a)^N$$

Taylor Series is just a Taylor Polynomial that goes on forever.

$f(x) = \sin x \to 0 \quad a = 0$ Maclurin

$f'(x) = \cos x \to 1$

$f''(x) = -\sin x \to 0$

$f'''(x) = -\cos \to -1$

$f''''(x) = \sin x \to 0$

$f^{\cancel{||||}}(x) = \cos x \to 1$

$$\sin = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \dots$$

MATH Taylor Series pt. 2

$f(x) = e^x \to e \quad a=1$

$f'(x) = e^x \to e$

$f''(x) = e^x \to e$

$f'''(x) = e^x \to e$

$f''''(x) = e^x - e$

$$e^x = e + e(x-1) + \frac{e}{2}(x-1)^2 + \frac{e}{3!}(x-1)^3 + \frac{e}{4!}(x-1)^4 + \dots$$

Thiels a short cut

4 Mallaulin Selies to know (only wolk for Maclaulin)

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \underset{\text{minus}}{+} \frac{x^7}{7!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!}$$

for all x

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \dots \text{ for } |x| < 1$$

Taylor Series Pt. 3

MATH

MACL. Series

$f(x) = x^3 e^x$

Just equals x^3 times e^x Maclaurin Series

$$= x^3\left(1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!}\right)$$

MACL. Series

$f(x) = \sin(x^2)$

$$\sin(\square) = \square - \frac{\square^3}{3!} + \frac{\square^5}{5!} - \frac{\square^7}{7!} + \dots$$

if You want $\sin x^2$ Just Put x^2 in the blank box

$$f(x) = \frac{1}{(1-x)^2} = \left(\frac{1}{1-x}\right)\left(\frac{1}{1-x}\right) = (1+x+x^2+x^3+x^4\dots)^2$$

$$\frac{d}{dx}\left(\frac{1}{1-x}\right) \to \frac{1}{(1-x)^2}$$

$$\frac{d}{dx}\left(1+x+x^2+x^3+x^4+\dots\right) = 1+2x+3x^2+4x^3+\dots$$

MATH Taylor Series Part 4

$F(x) = \text{Arctan}\, x$

$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \ldots$

replace x w $-x$ to get

$\frac{1}{1+x} = 1 - x - x^2 - x^3 - x^4 - \ldots$

replace x w x^2

$$\underbrace{\int \frac{1}{1+x^2}}_{\text{Arctan}\, x} = \int 1 - x^2 + x^4 - x^6 + x^8 \ldots$$

$\text{Arctan}\, x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7}$

$\arctan(1) = \frac{\pi}{4} \to 1 - \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9}$

$\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9}\right)$

$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$

$e^{ix} = 1 + ix + \frac{(ix)^2}{2} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \ldots$

wher $i = \sqrt{-1}$

$= 1 + ix - \frac{x^2}{2} - \frac{(ix)^3}{3!} + \frac{x^4}{4!} + \ldots$

$e^{ix} = \cos x \pm i \sin x$ → eulers formula

$$\boxed{e^{i\pi} = -1}$$