September 29, 2025

National Coffee Day

Today in History:

Willie Mays makes catch against the Cleveland Indians (1954)

Stacy Alison becomes the First American Woman to reach the summit of Mount Everest (1988)

Number of the Day: 4459

4459 = 7 x 7 x 7 x 13

4459 divides $18^{12} - 1$

Fun Fact:

The cruise liner QEII moves only 6 inches per gallon of diesel fuel.

Quote of the Day:

"Great things are done by a series of small things brought together."

Vincent Van Gogh

Today's Weather:

Sunny and warm, high 75^{o}

Math 121

Quiz #17

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for

$$y = x^2 e^x$$

$$\frac{dy}{dx} = x^2 e^x + 2x e^x$$

$$= (x^2 + 2x) e^x$$

$$\frac{d^2y}{dx^2} = (x^2 + 2x) e^x + (2x + 2) e^x$$

$$= (x^2 + 4x + 2) e^x$$

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mpod 17

(7)

$$f(x) = 2\sqrt{x} = 2x^{\frac{1}{2}}$$

$$f'(x) = x^{-\frac{1}{2}}$$

$$f''(x) = -\frac{1}{2} x^{-\frac{3}{2}}$$

$$f'''(x) = \frac{3}{4} x^{-\frac{5}{2}}$$

(29) $f(x) = x^{-1}$

$f'(x) = -x^{-2}$

$f''(x) = 2x^{-3}$

$f'''(x) = -6x^{-4}$

$f^{(4)}(x) = 24x^{-5}$

$f^{(5)}(x) = -120x^{-6}$

$f^{6}(x) = 720x^{-7}$

TRIG DERIVATIVES

(1) $f(x) = \sin x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{x}$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \cos(h) - \sin x}{h} + \lim_{h \to 0} \frac{\cos x \sin(h)}{h}$$

$$= \lim_{h \to 0} \sin x \left(\frac{\cos(h) - 1}{h} \right) + \lim_{h \to 0} \cos x \frac{\sin(h)}{h}$$

(the limit $\frac{\cos(h)-1}{h}$ goes to 0; the limit $\frac{\sin(h)}{h}$ goes to 1)

$$= \cos x$$

② $f(x) = \cos x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{\cos x \cos(h) - \sin x \sin(h) - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{\cos x(\cos(h) - 1)}{h} - \lim_{h \to 0} \frac{\sin x \sin(h)}{h}$$

$\cos x \cdot 0$ $\qquad$ $\sin x \cdot 1$

$$= -\sin x$$

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③ $f(x) = \tan x$

$$= \frac{\sin x}{\cos x}$$

$$f'(x) = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

(4) $f(x) = \cot x$

$$= \frac{\cos x}{\sin x}$$

$$f'(x) = \frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{(\sin x)^2}$$

$$= \frac{-1}{\sin^2 x} = -\csc^2 x$$

(5) $f(x) = \sec x$

$$= \frac{1}{\cos x}$$

$$f'(x) = \frac{(\cos x)(0) - (1)(-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$= \tan x \cdot \sec x$$

(6) $f(x) = \csc x$

$$= \frac{1}{\sin x}$$

$$f'(x) = \frac{(\sin x)(0) - (1)(\cos x)}{\sin^2 x}$$

$$= \frac{-\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}$$

$$= -\cot x \csc x$$

$f(x)$	$f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\sec x$	$\tan x \sec x$
$\csc x$	$-\cot x \csc x$

EXAMPLE 1

$$f(x) = x \sin x$$

$$f'(x) = x\cos x + \sin x$$

~~$\cos x \; x$~~

EXAMPLE 2 $\quad f(x) = \dfrac{x^2+x}{\cos x}$

$$f'(x) = \frac{(2x+1)\cos x - (x^2+x)(-\sin x)}{(\cos x)^2}$$

EXAMPLE 3 $\quad f(x) = 3\sec x - 10\cot x$

$$f'(x) = 3\sec x \tan x - 10(-\csc^2 x)$$

EXAMPLE 4

$$y = \frac{\sin x + \csc x}{e^x + x}$$

$$\frac{dy}{dx} = \frac{(e^x + x)(\cos x - \csc x \cot x) - (\sin x + \csc x)(e^x + 1)}{(e^x + x)^2}$$