October 27, 2025

National Black Cat Day

Today in History:

New York City subway opens (1904)

King George III speaks to Parliament of American rebellion (1775)

Number of the Day: 140

140 = 2 x 2 x 5 x 7

140 is a square pyramidal number.

Fun Fact:

Cyclophobia is the fear of bicycles.

Quote of the Day:

“I had rather be right than be President.”

-Henry Clay

Today’s Weather:

Bundant sunshine, high near 60°.

Math 121 - Quiz #29

For

$$f(x) = x^4 - 4x^3 - 15$$

Find where f is concave up, concave down and any inflection points.

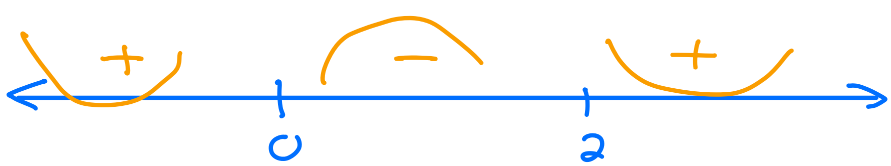
$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$

$12x^2 - 24x = 0$

$12x(x-2) = 0$

$x = 0, 2$



CON UP $(-\infty, 0) \cup (2, \infty)$

CON DN $(0, 2)$

I.P $x = 0, 2$

MPOD 29

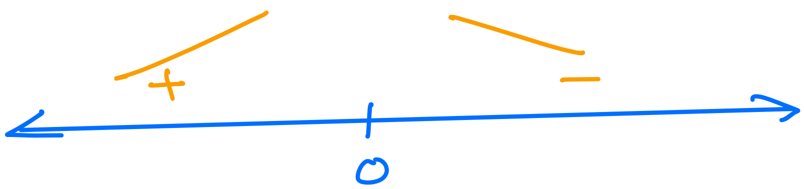
#3

$$f(x) = \frac{6}{x^2+3} = 6(x^2+3)^{-1}$$

$$f'(x) = -6(x^2+3)^{-2}(2x) = \boxed{\frac{-12x}{(x^2+3)^2}}$$

$$\frac{-12x}{(x^2+3)^2} = 0 \qquad -12x = 0 \qquad (x^2+3)^2 = 0$$

$$x = 0 \qquad \text{NONE}$$



INC: $(-\infty, 0)$ DEC $(0, \infty)$ C.P $x = 0$

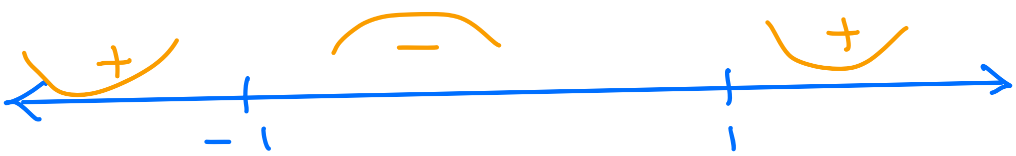
$$f''(x) = \frac{(x^2+3)^{\cancel{2}}(-12) - (-12x)(2)\cancel{(x^2+3)}(2x)}{(x^2+3)^{\cancel{4}\,3}}$$

$$= \frac{-12x^2 - 36 + 48x^2}{(x^2+3)^3} = \frac{36x^2 - 36}{(x^2+3)^3}$$

$$36x^2 - 36 = 0 \qquad (x^2+3)^3 = 0$$

$$x^2 - 1 = 0 \qquad \text{NONE}$$

$$x = \pm 1$$



CON UP $(-\infty, -1) \cup (1, \infty)$ CON DN $(-1, 1)$

IP $x = \pm 1$

L'Hospital's Rule.

① $\lim\limits_{x \to 2} \dfrac{x^2-4}{x-2} = \lim\limits_{x \to 2} \dfrac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}} = 4$

② $\lim\limits_{x \to 0} \dfrac{e^{2x}-1}{x} = \text{"}\dfrac{0}{0}\text{"}$

IF $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \text{"}\dfrac{0}{0}\text{"}$

LOOK AT $\lim\limits_{x \to a} \dfrac{f'(x)}{g'(x)} = L$

THEN $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = L$

L'HOSPITAL'S RULE

② $\lim\limits_{x \to 0} \dfrac{e^{2x}-1}{x} = \text{"}\dfrac{0}{0}\text{"}$

$\lim\limits_{x \to 0} \dfrac{2e^{2x}}{1} = \dfrac{2}{1} = 2$

$\lim\limits_{x \to 0} \dfrac{e^{2x}-1}{x} = 2$

③ $\lim\limits_{x \to 0} \dfrac{\sin(2x)}{e^{x}-1} = \text{"}\dfrac{0}{0}\text{"}$

$$\lim_{x\to 0} \frac{2\cos(2x)}{e^x} = \frac{2}{1} = 2$$

$$\lim_{x\to 0} \frac{\sin(2x)}{e^x - 1} = 2$$

(4) $$\lim_{x\to 1} \frac{1 - x + \ln x}{1 + \cos(\pi x)} = \frac{\text{"}0\text{"}}{0}$$

$$\lim_{x\to 1} \frac{-1 + \frac{1}{x}}{-\pi \sin(\pi x)} = \frac{\text{"}0\text{"}}{0}$$

$$\lim_{x\to 1} \frac{-\frac{1}{x^2}}{-\pi^2 \cos(\pi x)} = \frac{-1}{\pi^2}$$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\text{"}\infty\text{"}}{\infty} = L$$

$$\lim_{x\to a} \frac{f'(x)}{g'(x)} = L$$

(6) $$\lim_{x\to \infty} \frac{x^2 + 5x + 3}{3x^2 + 2x + 1} = \frac{\text{"}\infty\text{"}}{\infty} = \frac{1}{3}$$

$$\lim_{x\to \infty} \frac{2x + 5}{6x + 2} = \frac{\text{"}\infty\text{"}}{\infty}$$

$$\lim_{x\to \infty} \frac{2}{6} = \frac{1}{3}$$

$$\text{"}0\cdot\infty\text{"}$$

(7) $$\lim_{x\to 0^+} \underline{x^2}\,\underline{\ln x} = \text{"}0\cdot\infty\text{"} = 0$$

$$= \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x^2}} = \frac{\text{"}\infty\text{"}}{\infty}$$

$$\lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-2}{x^3}} = \lim_{x\to 0^+} \frac{1}{\cancel{x}}\,\frac{x^{\cancel{3}\,2}}{-2} = 0$$

$$\infty - \infty$$

(8) $$\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{\sin x}\right) = \text{"}\infty - \infty\text{"} = 0$$

$$\lim_{x\to 0^+} \frac{\sin x - x}{x\sin x} = \text{"}\frac{0}{0}\text{"}$$

$$\lim_{x\to 0^+} \frac{\cos x - 1}{x\cos x + \sin x} = \text{"}\frac{0}{0}\text{"}$$

$$\lim_{x\to 0^+} \frac{-\sin x}{-x\sin x + \cos x + \cos x} = \frac{0}{2} = 0$$

"0^0"

(9) $$\lim_{x\to 0^+} x^x = 1$$

$$\ln y = \ln \lim_{x\to 0^+} x^x$$

$$= \lim_{x\to 0^+} \ln x^x$$

$$= \lim_{x\to 0^+} x \ln x = \text{"}0\cdot\infty\text{"}$$

$$= \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = \text{"}\frac{\infty}{\infty}\text{"}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} \frac{1}{\cancel{x}}\left(\frac{-x^{\cancel{2}}}{1}\right) = 0$$

$$\ln y = 0$$

$$y = e^0 = 1$$

"1^∞"

(10) $$\lim_{x\to\infty} \left(1+\frac{2}{x}\right)^x = \text{"}1^\infty\text{"} = e^2$$

$$\underline{\ln y} = \ln \lim_{x\to\infty} \left(1+\frac{2}{x}\right)^x$$

$$= \lim_{x\to\infty} \ln\left(1+\frac{2}{x}\right)^x$$

$$= \lim_{x\to\infty} x \ln\left(1+\frac{2}{x}\right) = \text{"}0\cdot\infty\text{"}$$

$$= \lim_{x\to\infty} \frac{\ln\left(1+\frac{2}{x}\right)}{\frac{1}{x}} = \text{"}\frac{0}{0}\text{"}$$

$$\lim_{x\to\infty} \frac{\frac{1}{1+\frac{2}{x}}\left(+\frac{2}{\cancel{x^2}}\right)}{\left(+\frac{1}{\cancel{x^2}}\right)} = \underline{2}$$

$$\ln y = 2 \qquad y = e^2$$

TYPE I "$\frac{0}{0}$" "$\frac{\infty}{\infty}$" L'HOSPITAL

TYPE II "$0\cdot\infty$" "$\infty-\infty$"
USE ALGEBRA THEN L'HOSPITAL

TYPE III "0^0" "1^∞" "∞^0"
NATURAL LOG THE L'HOSPITAL