October 24, 2025

National Bologna Day

Today in History:

First barrel ride down Niagara Falls (1901)

The United Nations is born (1945)

Number of the Day: 756

756 = 2 x 2 x 3 x 3 x 3 x 7

756 is a pronic number, the product of two consecutive numbers, 27 and 28.

Fun Fact:

New Mexico is known as the Land of Enchantment.

Quote of the Day:

"Always do sober what you said you'd do drunk. It will teach you to keep your mouth shut."

- Ernest Hemingway

Today's Weather:

Windy, cloudy with showers likely, high near 50°.

Math 121 - Quiz #28

For

$$f(x) = \frac{1}{3}x^{4/3} - \frac{4}{3}x^{1/3}$$

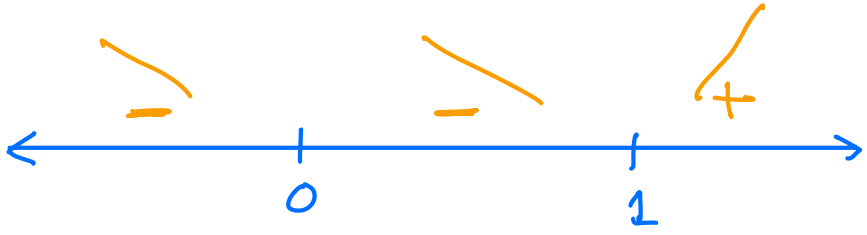
find where f is increasing, where f is decreasing and determine if the critical points are local maxima or minima.

$$f'(x) = \frac{4}{9}x^{\frac{1}{3}} - \frac{4}{9}x^{-\frac{2}{3}}$$

$$= \frac{4}{9}\left(x^{\frac{1}{3}} - x^{-\frac{2}{3}}\right) = \frac{4}{9}\left(x^{\frac{1}{3}} - \frac{1}{x^{2/3}}\right)$$

$$= \frac{4}{9}\left(\frac{x-1}{x^{2/3}}\right)$$

$x-1=0 \quad x=1$ UND $x^{2/3}=0 \quad x=0$



INC: $(1,\infty)$

DEC: $(-\infty,0)\cup(0,1)$

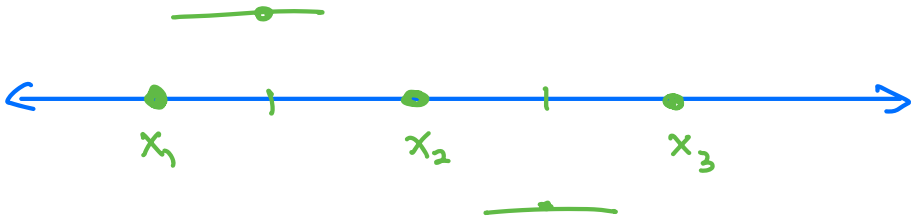
C.P. $x=0$ NOTHING

$x=1$ MIN.

MPOD

#2

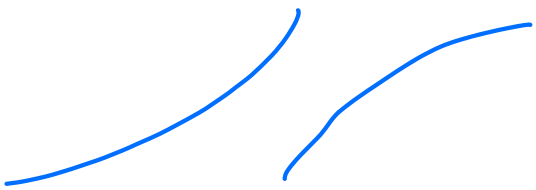
TRUE



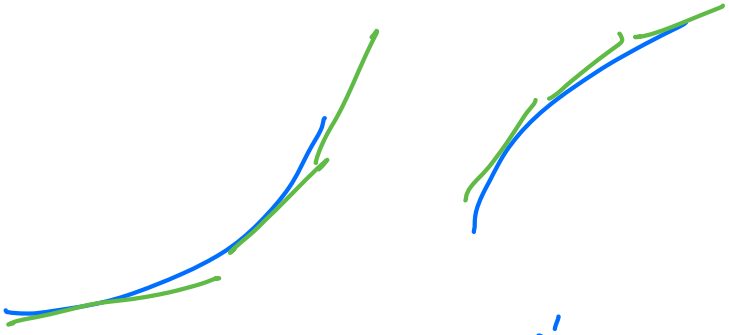
IF $f'(x) > 0$ $f(x)$ IS INCREASING

$f'(x) < 0$ $f(x)$ IS DECREASING

INC



DEC



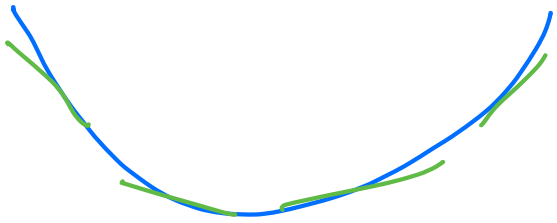
$f'(x)$ IS INC.

$f''(x) > 0$

$f'(x)$ IS DEC

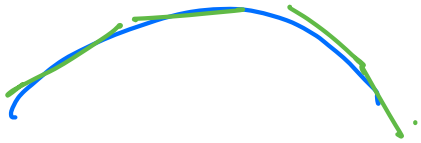
$f''(x) < 0$

IF $f''(x) > 0$



CONCAVE UP

IF $f''(x) < 0$



CONCAVE DN.

EXAMPLE 1

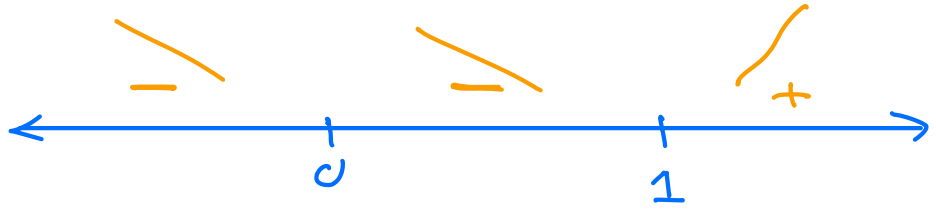
$$f(x) = 3x^4 - 4x^3 + 1$$

INC., DEC., C.P., CON UP, CON DN,

$$f'(x) = 12x^3 - 12x^2$$

$$12x^3 - 12x^2 = 0$$

$$12x^2(x-1) = 0 \qquad x = 0, 1$$



INC: $(1, \infty)$ DEC. $(-\infty, 0) \cup (0, 1)$

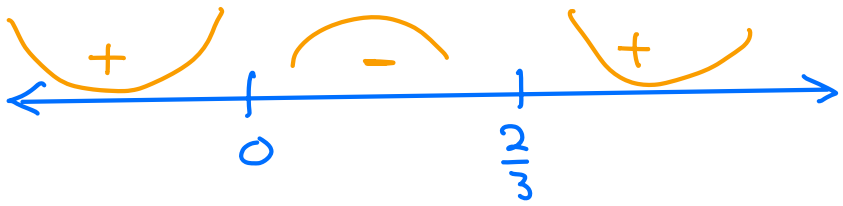
C.P. $x = 0$ NOTHING

$x = 1$ MIN.

$f''(x) = 36x^2 - 24x$

$36x^2 - 24x = 0$

$12x(3x - 2) = 0$ $\quad x = 0, \frac{2}{3}$



CON UP $(-\infty, 0) \cup (\frac{2}{3}, \infty)$

CON DN $(0, \frac{2}{3})$

INFLECTION POINT IS A POINT IN THE DOMAIN WHERE THE CONCAVITY CHANGES

$x = 0, \frac{2}{3}$ I.P.

EXAMPLE 2 $\quad f(x) = \dfrac{2(x^2-9)}{x^2-4}$ ←

INC. DEC. CP. → f' $\qquad$ CON UP CON DN IP. → f''

$$f'(x) = \frac{(x^2-4)(4x) - (2)(x^2-9)(2x)}{(x^2-4)^2}$$

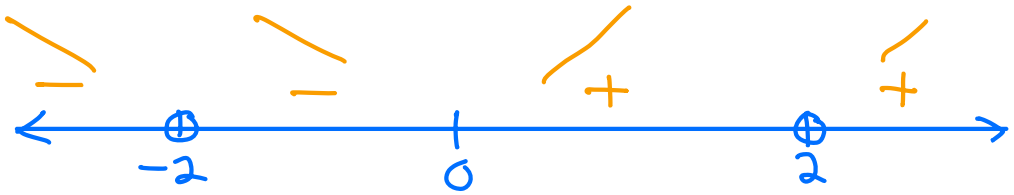
$$= \frac{4x^3 - 16x - 4x^3 + 36x}{(x^2-4)^2} = \frac{20x}{(x^2-4)^2}$$

$=0 \quad 20x=0$

$x=0$

UND. $(x^2-4)^2=0$

$x=\pm 2$



INC: $(0,2)\cup(2,\infty)$

DEC: $(-\infty,-2)\cup(-2,0)$

C.P. $x=0$ MIN

$$f''(x)=\frac{(x^2-4)^2(20)-(20x)(2)(x^2-4)(2x)}{(x^2-4)^{4\,3}}$$

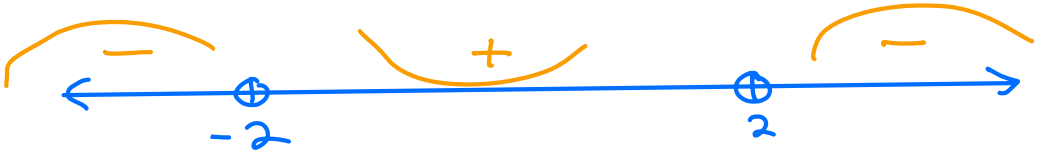
$$=\frac{20x^2-80-80x^2}{(x^2-4)^3}=\frac{-60x^2-80}{(x^2-4)^3}$$

$(x^2-4)^3=0$

$x=\pm 2$

$=0 \quad -60x^2-80=0$

NONE



CON UP $(-2,2)$

CON DN $(-\infty,-2)\cup(2,\infty)$

I.P. NONE

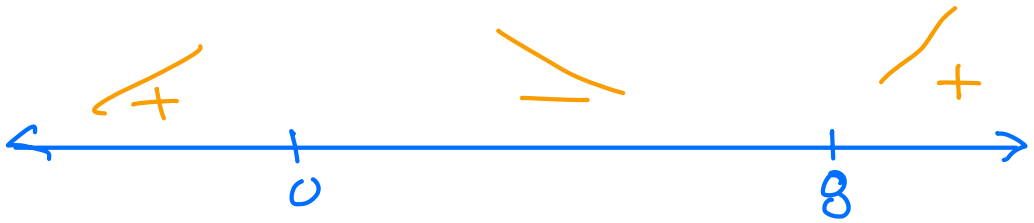
EXAMPLE

$f(x) = 2x^{\frac{5}{3}} - 5x^{\frac{4}{3}}$

$f'(x) = \frac{10}{3}x^{\frac{2}{3}} - \frac{20}{3}x^{\frac{1}{3}} = \frac{10}{3}x^{\frac{1}{3}}(x^{\frac{1}{3}} - 2)$

$=0$ $\quad x^{\frac{1}{3}}(x^{\frac{1}{3}} - 2) = 0$

$x = 0 \quad x^{\frac{1}{3}} - 2 = 0 \quad x = 8$



INC: $(-\infty, 0) \cup (8, \infty)$

DEC: $(0, 8)$

C.P. $x = 0$ MAX

$x = 8$ MIN

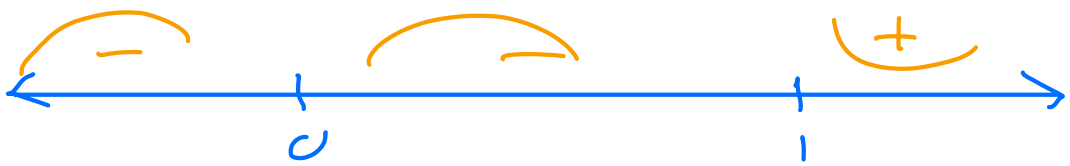
$f''(x) = \frac{20}{9}\left(\frac{x^{\frac{1}{3}} - 1}{x^{\frac{2}{3}}}\right)$

$=0$ $\quad x^{\frac{1}{3}} - 1 = 0$

$x = 1$

UND $\quad x^{\frac{2}{3}} = 0$

$x = 0$



CON UP $\quad (1, \infty)$

CON. DN $\quad (-\infty, 0) \cup (0, 1)$

I.P: $x = 1$