October 23, 2026

National Mole Day

Today in History:

Brutus commits suicide (42)

Chicago has its first #1 hit with “If You Leave Me Now” (1976)

Number of the Day: 217

217 = 7 x 31

217 is the ninth centered hexagonal (hex) number.

Fun Fact:

Americans spend approximately $25 billion each year on beer.

Quote of the Day:

“To get your playing more forceful, hit the drums harder.”

— Keith Moon (The Who)

Today’s Weather:

Plenty of sunshine. High 65^{o}

Math 121 - Quiz #28

For

$$f(x) = \frac{4}{3}x^{1/3} - \frac{1}{3}x^{4/3}$$

find where f is increasing, where f is decreasing and determine if the critical points are local maxima or minima.

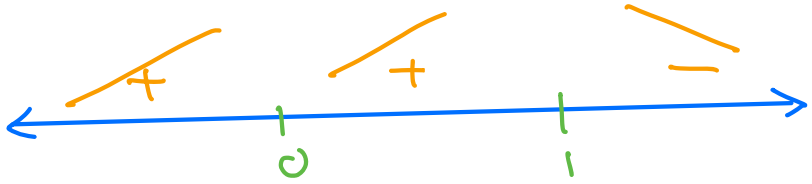
$$f'(x) = \frac{4}{9}x^{-\frac{2}{3}} - \frac{4}{9}x^{\frac{1}{3}}$$

$$= \frac{4}{9}\left(\frac{1}{x^{\frac{2}{3}}} - \frac{x^{\frac{1}{3}}}{1}\right)$$

$$= \frac{4}{9}\left(\frac{1-x}{x^{\frac{2}{3}}}\right)$$

$= 0 \quad 1 - x = 0 \quad x = 1$

$= \text{UND.} \quad x^{\frac{2}{3}} = 0 \quad x = 0$



INC: $(-\infty, 0) \cup (0, 1)$ $(-\infty, 1)$

DEC: $(1, \infty)$

CP.

$x = 0$ NOTHING.

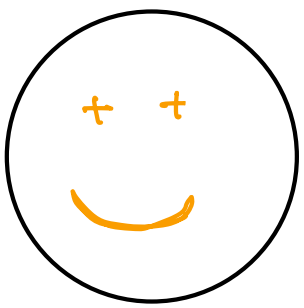
$x = 1$ LOCAL MAX

CONCAVITY



CONCAVE UP





$f''(x) > 0$

CONCAVE DN





$f''(x) < 0$

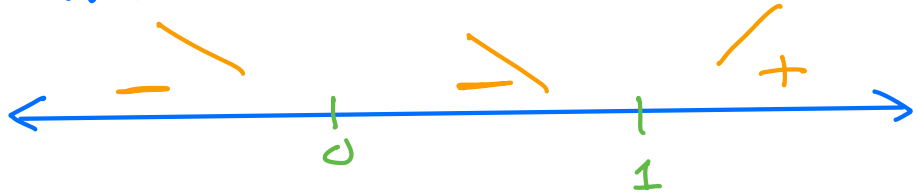
EXAMPLE $f(x) = 3x^4 - 4x^3 + 1$

INC / DEC / CON UP / CON DN

$f'(x) = 12x^3 - 12x^2$

$12x^3 - 12x^2 = 0$

$12x^2(x-1) = 0$ $\quad x = 0, 1$



INC: $[1,\infty)$ DEC: $(-\infty,0)\cup(0,1)$

$x=0$ NOTHING

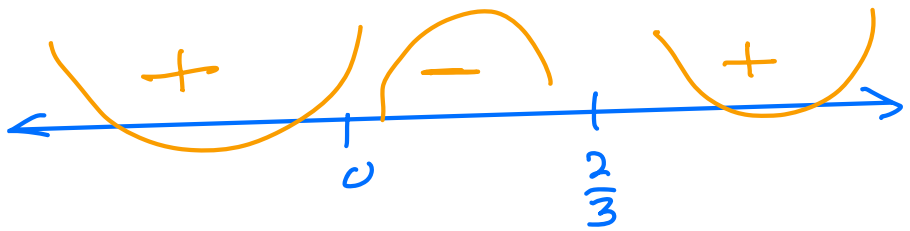
$x=1$ MIN

CON UP / CON DN

$f''(x) = 36x^2 - 24x$

$36x^2 - 24x = 0$

$12x(3x-2)=0$ $\quad x=0, \frac{2}{3}$



CON UP $(-\infty,0)\cup(\frac{2}{3},\infty)$

CON DN $(0,\frac{2}{3})$

INFLECTION POINT IS WHERE THE CONCAVITY CHANGES!!

$x=0, \quad x=\frac{2}{3}$

EXAMPLE 2 $\quad f(x) = \dfrac{2(x^2-9)}{x^2-4}$

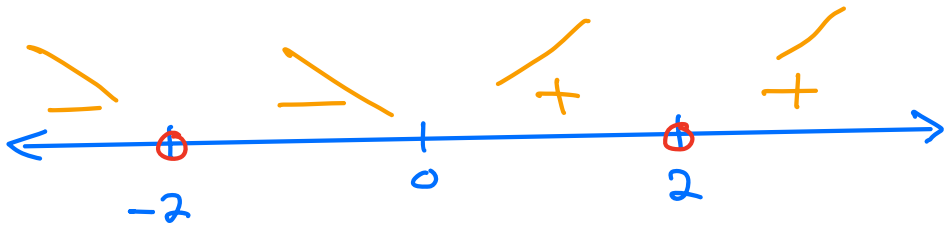
INC / DEC | CON UP | CON DN / C.P / I.P

$$f'(x) = \frac{(x^2-4)(4x) - (2)(x^2-9)(2x)}{(x^2-4)^2}$$

$$= \frac{20x}{(x^2-4)^2}$$

$= 0$ $\quad 20x = 0$ $\quad x = 0$

UND. $\quad (x^2-4)^2 = 0$ $\quad x = \pm 2$



INC: $(0,2) \cup (2,\infty)$ $\quad x=0$ MIN

DEC: $(-\infty,-2) \cup (-2,0)$

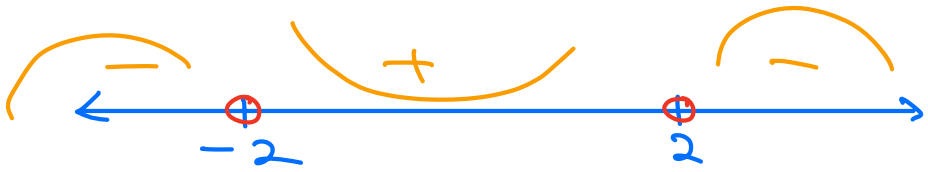
$$f'(x) = \frac{20x}{(x^2-4)^2}$$

$$f''(x) = \frac{(x^2-4)^2(20) - (20x)(2)(x^2-4)(2x)}{(x^2-4)^{4\,3}}$$

$$= \frac{-20(3x^2+4)}{(x^2-4)^3}$$

$= 0$ $\quad -20(3x^2+4) = 0$ $\quad$ NO!!

UND $\quad (x^2-4)^3 = 0$ $\quad x = \pm 2$



CON UP $(-2, 2)$

CON DN $(-\infty, -2) \cup (2, \infty)$

$$f(x) = 2x^{\frac{5}{3}} - 5x^{\frac{4}{3}}$$

$$\rightarrow f'(x) = \frac{10}{3}x^{\frac{2}{3}} - \frac{20}{3}x^{\frac{1}{3}}$$

$$= \frac{10}{3}x^{\frac{1}{3}}\left(x^{\frac{1}{3}} - 2\right)$$

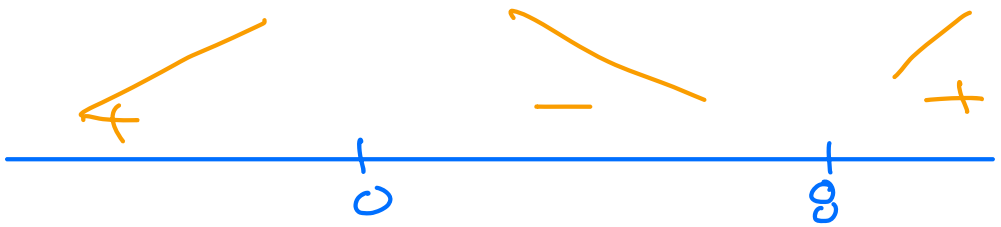
$$\frac{10}{3}x^{\frac{1}{3}}\left(x^{\frac{1}{3}} - 2\right) = 0$$

$= 0$

$x^{\frac{1}{3}} = 0 \qquad x^{\frac{1}{3}} - 2 = 0$

$x^{\frac{1}{3}} = 2$

$x = 8$



INC: $(-\infty, 0) \cup (8, \infty)$ CP $x = 0$ MAX

$x = 8$ MIN

DEC: $(0, 8)$

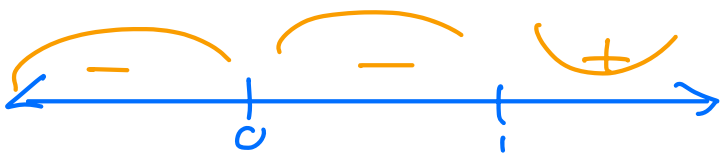
$$f''(x) = \frac{20}{9}x^{-\frac{1}{3}} - \frac{20}{9}x^{-\frac{2}{3}}$$

$$= \frac{20}{9}\left(\frac{1}{x^{\frac{1}{3}}} - \frac{1}{x^{\frac{2}{3}}}\right) = \frac{20}{9}\left(\frac{x^{\frac{1}{3}} - 1}{x^{\frac{2}{3}}}\right)$$

$x^{\frac{1}{3}} - 1 = 0 \qquad x = 1$

$= 0$

UND $x^{\frac{2}{3}} = 0 \qquad x = 0$



CON UP $(1, \infty)$

CON GIÙ $(-\infty, 0) \cup (0, 1)$

I.P. $x = 1$