

September 22, 2026

Ice Cream Cone Day

Today in History:

Lincoln issues Emancipation Proclamation (1862)

Nathan Hale executed for spying (1776)

Number of the Day: 5545

5545 = 5×1109

5545 uses only the digits 1 and 2 in base 4 (1112221)
and base 7 (22111).

Fun Fact:

Antarctica's area code is 672.

Quote of the Day:

"When the President does it, that means that it's not illegal."

Richard M. Nixon

Today's Weather:

Cloudy and breezy with a passing shower or two. High 65°

Math 121

Quiz #14

Find $f'(x)$ for

$$f(x) = \frac{x^2 + 4\sqrt{x}}{x^2} - e^{x+1}$$

$$f(x) = 1 + 4x^{-\frac{3}{2}} - e^x e^1$$

$$f'(x) = 0 + 4\left(-\frac{3}{2}\right)x^{-\frac{5}{2}} - e^x e^1$$

$$f'(x) = -6x^{-\frac{5}{2}} - e^{x+1}$$

PRODUCT RULE

$$P(x) = f(x) \cdot g(x)$$

$$P'(x) = \lim_{h \rightarrow 0} \frac{P(x+h) - P(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x) \cdot g(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x+h)g(x) + f(x+h)g(x) - f(x)g(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)}{h} \frac{(g(x+h) - g(x))}{h} + \lim_{h \rightarrow 0} \frac{g(x)}{h} \frac{(f(x+h) - f(x))}{h}$$

$$= f(x) \cdot g'(x) + g(x) f'(x)$$

$$p(x) = f(x) \cdot g(x)$$

$$p'(x) = f(x)g'(x) + g(x)f'(x)$$

EXAMPLES

$$f(x) = (2x+4)(3x^2+2x+1)$$

$$f'(x) = (2x+4)(6x+2) + (3x^2+2x+1)(2)$$

$$f(x) = (e^x + x^2)\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$$

$$f'(x) = (e^x + x^2)\left(\frac{1}{2}x^{-\frac{1}{2}} + -\frac{1}{2}x^{-\frac{3}{2}}\right)$$

$$+ \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)(e^x + 2x)$$

$$f(x) = \underbrace{e^x}_{\text{}} \underbrace{(x^2+1)(x^3+1)}_{\text{}}$$

$$f'(x) = e^x \left[(x^2+1)(3x^2) + (x^3+1)(2x) \right] + (x^2+1)(x^3+1)e^x$$

$$f(x) = P(x) Q(x) R(x)$$

$$f'(x) = P(x) Q(x) R'(x) + P(x) Q'(x) R(x) + P'(x) Q(x) R(x)$$

QUOTIENT RULE

$$q(x) = \frac{f(x)}{g(x)}$$

$$q'(x) = \lim_{h \rightarrow 0} \frac{q(x+h) - q(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x) - f(x)g(x+h)}{h g(x+h)g(x)}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x) - \cancel{f(x)g(x)} + \cancel{f(x)g(x)} - f(x)g(x+h)}{h g(x+h)g(x)}$$

$$\lim_{h \rightarrow 0} \frac{g(x) \left(\frac{f(x+h) - f(x)}{h} \right) - f(x) \left(\frac{g(x+h) - g(x)}{h} \right)}{g(x+h)g(x)}$$

$$= \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$g(x) = \frac{f(x)}{g(x)}$$

$$g'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$g(x) = \frac{H_1}{H_0}$$

$$g'(x) = \frac{H_0 dH_1 - H_1 dH_0}{H_0 H_0}$$

$$f(x) = \frac{x^2 + 1}{3x + 5}$$

$$f'(x) = \frac{(3x+5)(2x) - (x^2+1)(3)}{(3x+5)^2}$$

$$f(x) = \frac{\sqrt{x}}{e^x}$$

$$f'(x) = \frac{e^x \left(\frac{1}{2} x^{-\frac{1}{2}} \right) - (\sqrt{x}) e^x}{(e^x)^2}$$

$$f(x) = \frac{(x^2+x+1)(3x^5+4)}{(x+2)}$$

$$f'(x) = \frac{(x+2) \left[(x^2+x+1)(15x^4) + (3x^5+4)(2x+1) \right] - \left[(x^2+x+1)(3x^5+4) \right] (1)}{(x+2)^2}$$

RECIP. RULE

$$f(x) = \frac{1}{g(x)}$$

$$f'(x) = \frac{\cancel{(g(x))} (0) - 1 g'(x)}{(g(x))^2} = - \frac{g'(x)}{(g(x))^2}$$

EXAMPLE

$$f(x) = \frac{1}{x^3+x^2+x+1}$$

$$f'(x) = - \frac{3x^2+2x+1}{(x^3+x^2+x+1)^2}$$