

September 18, 2024

National Cheeseburger Day

Today in History:

Washington lays the cornerstone to the Capitol (1793)

Patty Hearst is captured (1975)

Number of the Day: 854

854 = $2 \times 7 \times 61$

854 is the largest number that together with its square (729,316) uses all digits not more than once

Fun Fact:

The Russians showed up 12 days late to the 1908 Olympics in London because they were using the Julian calendar instead of the Gregorian calendar.

Quote of the Day:

“Clothes make the man. Naked people have little or no influence on society.”

-Mark Twain

Today's Weather:

Partly cloudy and some sun, high 72°

Math 121

Quiz #12

Use the definition of derivative to find $f'(4)$ for

$$f(x) = \sqrt{x}$$

$$f'(4) = \lim_{h \rightarrow 0} \frac{f(4+h) - f(4)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{4+h} - \sqrt{4}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{4+h} - 2}{h} \cdot \frac{\sqrt{4+h} + 2}{\sqrt{4+h} + 2}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{4} + \cancel{h} - \cancel{4}}{\cancel{h}(\sqrt{4+h} + 2)} = \frac{1}{2+2} = \frac{1}{4}$$

TEST 1

HIGH 98 (4)
 AVE 83.23
 MED 85

90-100	108
80-89	201
70-79	70
60-69	26
50-59	7
↓	4

1a) $a = -1$ $b = 4$ $-1 \leq x \leq 4$

b) 6

c) $y - 2 = \frac{3}{2}(x - 3)$

d) $\sin\left(x + \frac{1}{x}\right)$

2a) i) $\cot \theta = \frac{1}{\sqrt{5}}$

ii) $\cos \theta = -\frac{1}{\sqrt{6}}$

b) $x = 3, -1$

c) ~~3, 4~~

3a) 2α

b) 6

c) $-\frac{6}{5}$

$$e^{-1} \leq e^{\sin(\frac{1}{x})} \leq e$$

$$e^{-1/x^2} \leq x^2 e^{\sin(\frac{1}{x})} \leq x^2 e$$

$\downarrow$ $\rightarrow 0$ $\downarrow$ $\rightarrow 0$
 $= 0$

4a)

$$f'(x) = \frac{4x}{x-2}$$

b)

$$\lim_{x \rightarrow 0}$$

$$\frac{\frac{3x}{x} + \frac{4 \sin 3x \cdot 3}{3x}}{5 \frac{\sin 5x}{5x} - \frac{x \cos 2x}{x}} = \frac{3+12}{5-1} = \frac{15}{4}$$

c)

$$\frac{\sqrt{3}}{3}$$

d)

$$a=2$$

5)

T F T F T F T F T F

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

CONSTANT RULE

$$f(x) = C$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{C - C}{h} = 0$$

POWER RULE

$$f(x) = x^N$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^N - x^N}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{x^N} + N x^{N-1} \cancel{h} + \left(\frac{N(N-1)}{2} \right) x^{N-2} h^2 + \dots + h^N - \cancel{x^N}}{h}$$

$$= N x^{N-1}$$

$$f(x) = x^N$$

$$f'(x) = N x^{N-1}$$

CONSTANT MULTIPLE RULE

$$f(x) = c g(x)$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{c g(x+h) - c g(x)}{h}$$

$$= c \boxed{\lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h}}$$

$$= c g'(x)$$

SUM RULE

$$f(x) = g(x) + \underline{k(x)}$$

DIFF. RULE

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{g(x+h) + \underline{k(x+h)} - (g(x) + \underline{k(x)})}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{g(x+h) - g(x)}{h} + \frac{\underline{k(x+h)} - \underline{k(x)}}{h}}{1}$$

$$g'(x) + \underline{k'(x)}$$

EXPONENTIAL RULE $f(x) = b^x$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{b^{x+h} - b^x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{b^x b^h - b^x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{b^x (b^h - 1)}{h}$$

$$= b^x \lim_{h \rightarrow 0} \frac{b^h - 1}{h}$$

$$b=2 \quad \lim_{h \rightarrow 0} \frac{b^h - 1}{h} < 1$$

$$b=3 \quad \lim_{h \rightarrow 0} \frac{b^h - 1}{h} > 1$$

$$b = 2.718281828459 \dots = e$$

$$\lim_{h \rightarrow 0} \frac{b^h - 1}{h} = 1$$

$$f'(x) = b^x \lim_{h \rightarrow 0} \frac{b^h - 1}{h} = b^x$$

$$f(x) = e^x$$

$$f'(x) = e^x$$