September 17, 2025

National Apple Dumpling Day

Today in History:

U.S. Constitution signed (1787)

Space Shuttle unveiled (1976)

Number of the Day: 579

579 = 3 x 193

579 is a sum of the squares of three consecutive primes.

Fun Fact:

Didaskaleinophobia is the fear of school.

Quote of the Day:

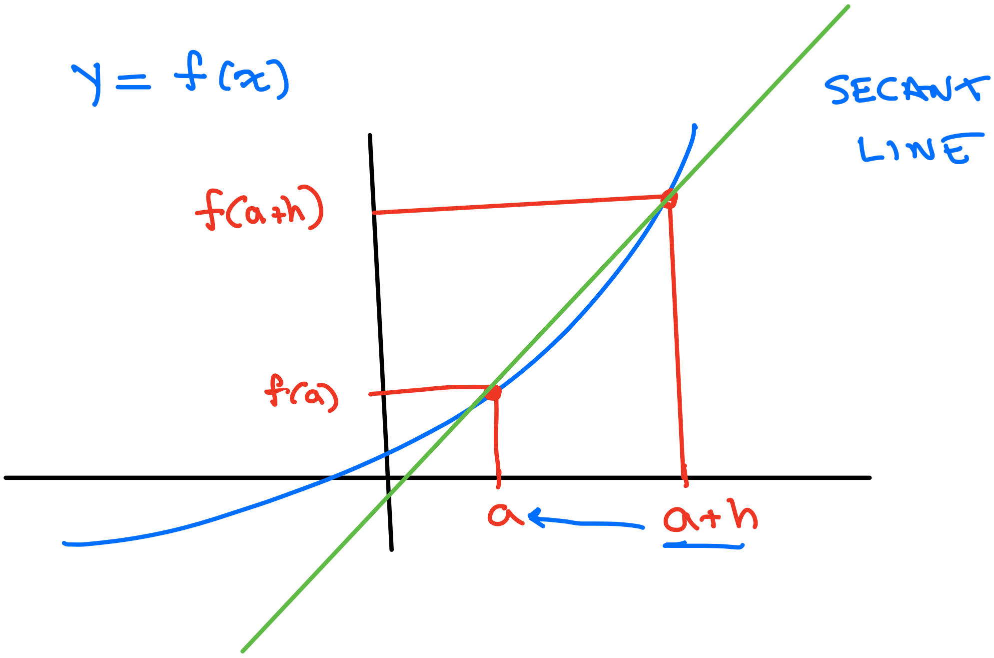
“I have always been afraid of banks.”

Andrew Jackson

Today’s Weather:

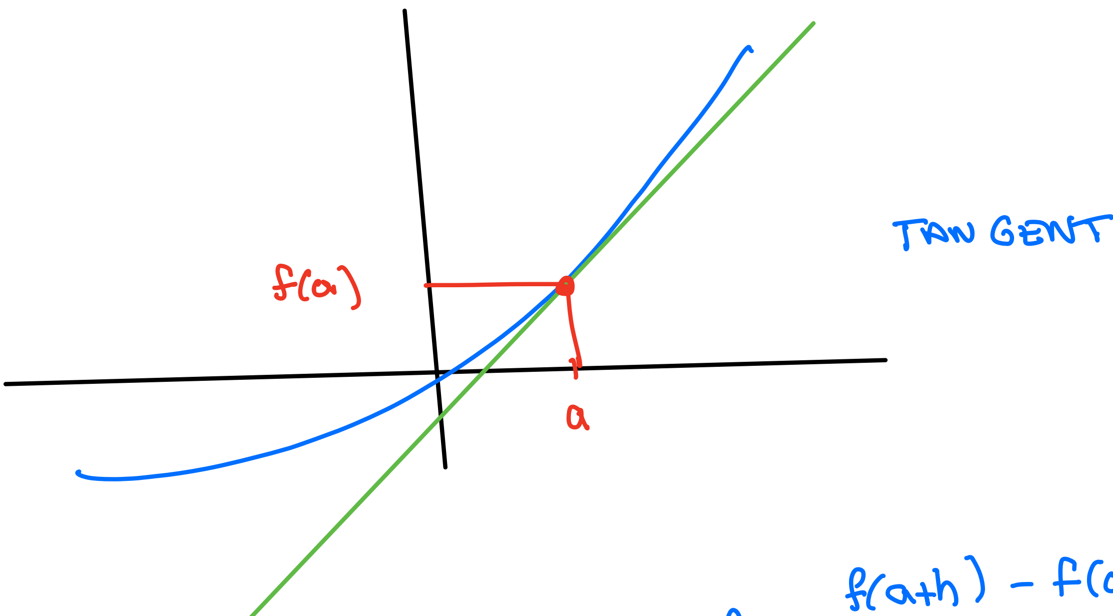
Sunny, high near 76^{o}

$y = f(x)$



$$\text{SLOPE} = \frac{\Delta y}{\Delta x} = \frac{f(a+h) - f(a)}{\cancel{a} + h - \cancel{a}}$$

$$= \frac{f(a+h) - f(a)}{h}$$



SLOPE TANGENT LINE $\lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

THE DERIVATIVE

THE DERIVATIVE IS THE SLOPE OF THE TANGENT LINE

EXAMPLE 1 $\qquad \underline{f(x) = 3} \qquad a = 2$

$$f'(2) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{3 - 3}{h} = 0$$

EXAMPLE 2 $\qquad f(x) = c \qquad x = a$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{c - c}{h} = 0$$

EXAMPLE 3 $\qquad \underline{f(x) = mx + b} \qquad x = a$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{m(a+h) + b - (ma+b)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{ma} + m\cancel{h} + \cancel{b} - \cancel{ma} - \cancel{b}}{\cancel{h}}$$

$$\underline{= m}$$

<u>EXAMPLE</u> 4 $\qquad f(x) = x^2 \qquad x = a$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{(a+h)^2 - a^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{a^2} + 2a\cancel{h} + h^{\cancel{2}} - \cancel{a^2}}{\cancel{h}}$$

$$= \lim_{h\to 0} 2a + h = 2a$$

$f(x) = x^2$ $\quad\quad f'(a) = 2a$

EXAMPLE 5 $\quad f(x) = x^3 \quad\quad x = a$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{(a+h)^3 - a^3}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{a^3} + 3a^2\cancel{h} + 3ah^{\cancel{2}} + h^{\cancel{3}\,2} - \cancel{a^3}}{\cancel{h}}$$

$$= \lim_{h\to 0} 3a^2 + 3ah + h^2 = 3a^2$$

$f(x) = x^3$ $\quad\quad f'(a) = 3a^2$

$f(x) = x^2$ $\quad\quad f'(a) = 2a$

EXAMPLE 6 $\quad f(x) = \frac{1}{x} \qquad x = a$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{a+h} - \frac{1}{a}}{h}$$

$$\rightarrow \quad = \lim_{h \to 0} \frac{\cancel{a} - (\cancel{a} + \cancel{h})^{-1}}{(a+h)(a)\cancel{h}}$$

$$= \lim_{h \to 0} \frac{-1}{(a+h)(a)} = \frac{-1}{a^2} = -a^{-2}$$

$$f(x) = \frac{1}{x} = x^{-1} \qquad f'(a) = -a^{-2}$$

$f(x) = \sqrt{x} \qquad x = a$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{a+h} - \sqrt{a}}{h} \left(\frac{\sqrt{a+h} + \sqrt{a}}{\sqrt{a+h} + \sqrt{a}} \right)$$

$$= \lim_{h \to 0} \frac{\cancel{a} + \cancel{h}^{\,1} - \cancel{a}}{\cancel{h}(\sqrt{a+h} + \sqrt{a})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{a+h}+\sqrt{a}} = \frac{1}{2\sqrt{a}}$$

$$f(x) = \sqrt{x} = x^{\frac{1}{2}} \qquad f'(a) = \frac{1}{2\sqrt{a}} = \frac{1}{2}a^{-\frac{1}{2}}$$

$$f(x) = \sqrt{x} \qquad f'(a) = \frac{1}{2\sqrt{a}}$$

$x = 4$ SLOPE OF THE TANGENT LINE?

$$f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

EQUATION OF TANGENT LINE

$$(4, \sqrt{4}) = (4, 2) \qquad m = \frac{1}{4}$$

$$y - 2 = \frac{1}{4}(x-4)$$

$$f(x) = x^2 \qquad f'(a) = 2a \qquad \boxed{f'(x) = 2x}$$

DERIVATIVE FUNCTION
GIVES SLOPE OF TANGENT LINE

$f(x) = mx + b$ $\quad$ $f'(x) = m$

$f(x) = x^2$ $\quad$ $f'(x) = 2x$

$f(x) = x^3$ $\quad$ $f'(x) = 3x^2$

$f(x) = \frac{1}{x}$ $\quad$ $f'(x) = \frac{-1}{x^2}$

$f(x) = \sqrt{x}$ $\quad$ $f'(x) = \frac{1}{2\sqrt{x}}$