

October 15, 2025

Global Handwashing Day

Today in History:

"I love Lucy" made its debut (1951)

Marta Hari executed (1917)

Number of the Day: 1124

1124 = $2 \times 2 \times 281$

1124 is the maximum number of regions into which 34 circles divide a plane.

Fun Fact:

Donald Duck's middle name is Fauntleroy.

Quote of the Day:

"You never run out of things that can go wrong."

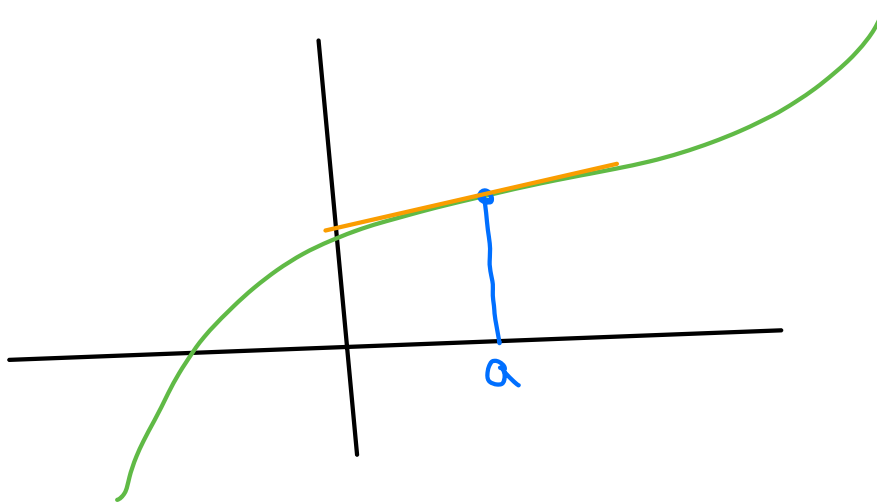
- Edward A. Murphy

Today's Weather:

A shower or two around the area early, partly cloudy in the afternoon,

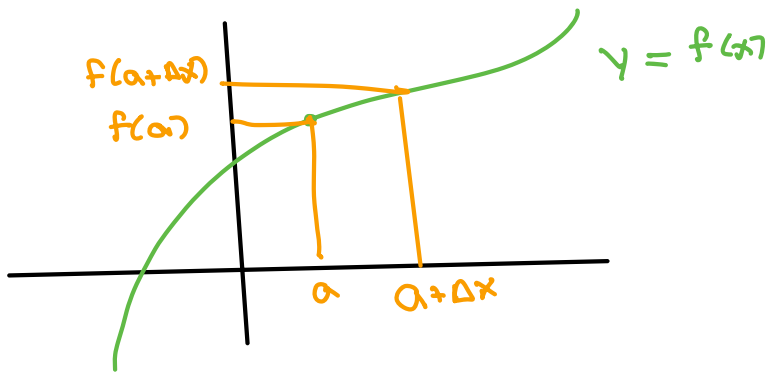
high 61°

LINEAR APPROXIMATIONS



$$\frac{dy}{dx} = f'(x)$$

SLOPE
OF
TANGENT
LINE



$$\Delta y = f(a + \Delta x) - f(a)$$

IF Δx IS "SMALL" THEN

$$\frac{\Delta y}{\Delta x} \approx f'(x)$$

$$\Delta y \approx f'(x) \Delta x$$

Example 1

$$\text{Approx } \frac{1}{1.1} - \frac{1}{1}$$

$$f(x) = \frac{1}{x} \quad a = 1 \quad \Delta x = .1$$

$$\frac{1}{1.1} - \frac{1}{1} = f(1.1) - f(1) \approx f'(1)(.1)$$

$$f'(x) = -\frac{1}{x^2} \quad f'(1) = -1$$

$$f'(1) \cdot (.1) = (-1)(.1) = -.1$$

$$\frac{1}{1.1} - \frac{1}{1} \approx -.1$$

$$\frac{1}{1.1} - \frac{1}{1} = -.0909 \dots$$

Example 2

$$\sqrt{16.1} - \sqrt{16}$$

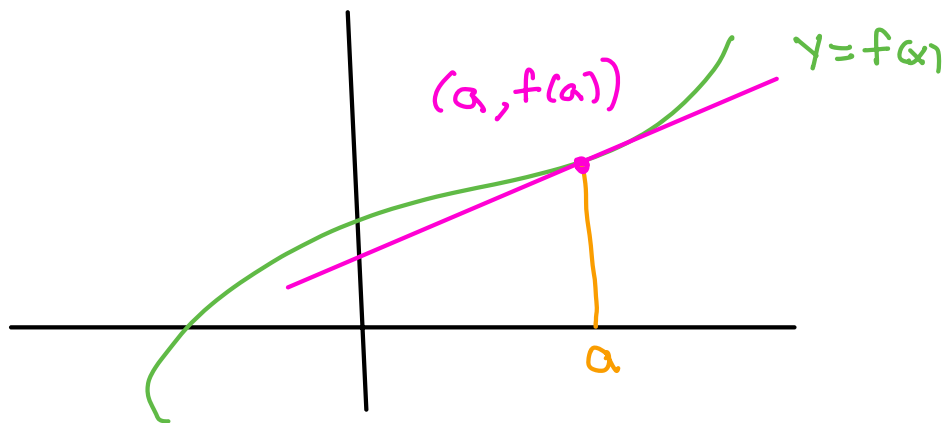
$$f(x) = \sqrt{x} \quad a = 16 \quad \Delta x = .1$$

$$\sqrt{16.1} - \sqrt{16} = f(16.1) - f(16) \approx f'(16)(.1)$$

$$f'(x) = \frac{1}{2\sqrt{x}} \quad f'(16) = \frac{1}{2\sqrt{16}} = \frac{1}{8} = .125$$

$$\sqrt{16.1} - \sqrt{16} \approx (.125)(.1) = .0125$$

$$\sqrt{16.1} - \sqrt{16} = .01248 \quad \leftarrow \text{😊}$$



EQUATION OF TANGENT LINE:

$(a, f(a))$ SLOPE $f'(a)$

$$y - f(a) = f'(a)(x - a)$$

$$y = f(a) + f'(a)(x - a)$$

$$L(x) = f(a) + f'(a)(x - a)$$

LOCAL LINEAR APPROX.

IF x IS "CLOSE" TO a

$$L(x) \approx f(x)$$

EXAMPLE

$$f(x) = x^2$$

$$a = 4$$

$$L(x) = f(a) + f'(a)(x - a)$$

$$f(4) = 16$$

$$f'(x) = 2x$$

$$f'(4) = 8$$

$$L(x) = 16 + 8(x - 4)$$

$$L(x) = 16 + 8(x-4)$$

$$\begin{aligned} L(4.1) &= 16 + 8(4.1-4) \\ &= 16 + 8(.1) = 16.8 \end{aligned}$$

$$L(4.1) \approx f(4.1) = (4.1)^2 = 16.81$$

$$\begin{aligned} L(3.8) &= 16 + 8(3.8-4) \\ &= 16 + 8(-.2) \\ &= 16 - 1.6 = 14.4 \end{aligned}$$

$$L(3.8) \approx f(3.8) = (3.8)^2 = 14.44$$

$$\begin{aligned} L(10) &= 16 + 8(10-4) \\ &= 16 + 8(6) \\ &= 16 + 48 = 64 \end{aligned}$$

$$L(10) \approx f(10) = (10)^2 = 100$$

BAD WORD
☹

$$f(x) = \ln x$$

$$a = 1$$

$$L(x) = f(a) + f'(a)(x-a)$$

$$f(1) = 0$$

$$L(x) = 0 + 1(x-1)$$

$$f'(x) = \frac{1}{x}$$

$$L(x) = (x-1)$$

$$f'(1) = 1$$

$$\ln(.9) \approx L(.9) = (.9 - 1) = -.1$$

$$\ln(.9) = -.105 \quad \Longleftrightarrow$$

$$\ln(1.3) \approx L(1.3) = (1.3 - 1) = .3$$

$$\ln(1.3) = .262$$