November 14, 2025

National Pickle Day

Today in History:

Moby Dick published (1851)

Plane crash devastates Marshall University Football Team (1970)

Number of the Day: 1729

1729 = $7 \times 13 \times 19$

1729 = $1^3 + 12^3 = 9^3 + 10^3$

Fun Fact:

Oscar Wilde’s last words were “Either the wallpaper goes or I do”.

Quote of the Day:

"Never interrupt your enemy when he is making a mistake."

- Napoleon Bonaparte

Today’s Weather:

Partly cloudy, high of 57^{o}.

Math 121 - Quiz #40

Find

$$\int x\,(x-1)^7\,dx$$

$u = x - 1 \qquad x = u + 1$

$du = dx$

$$= \int (u+1)\,u^7\,du = \int u^8 + u^7\,du$$

$$= \frac{u^9}{9} + \frac{u^8}{8} = \frac{(x-1)^9}{9} + \frac{(x-1)^8}{8} + C$$

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mPOD 40 #15

$$\int_0^{\sqrt{\pi}} x \sin(x^2)\,dx \qquad u = x^2 \qquad du = 2x\,dx \qquad dx = \frac{du}{2x}$$

$$= \int \cancel{x} \sin(u)\,\frac{du}{2\cancel{x}} = \frac{-1}{2}\cos u$$

$$= -\frac{1}{2}\cos(x^2)\Big|_0^{\sqrt{\pi}} = -\frac{1}{2}\left[-1 - 1\right] = 1$$

#2

$$\int (x+1)\sqrt{x}\,dx = \int \left(x^{\frac{3}{2}} + x^{\frac{1}{2}}\right) dx$$

$$= \frac{2x^{\frac{5}{2}}}{5} + \frac{2x^{\frac{3}{2}}}{3} + C$$

RECALL

$$\frac{d}{dx}\left(b^{x}\right) = b^{x}\ln b$$

$$\int b^{x}\,dx = \frac{b^{x}}{\ln b} + C$$

EXAMPLES:

1) $$\int 3^{x}\,dx = \frac{3^{x}}{\ln 3} + C$$

2) $$\int 7^{-x}\,dx$$ $u = -x$, $du = -dx$ $$-\int 7^{u}\,du$$

$$= -\frac{7^{u}}{\ln 7} = -\frac{7^{-x}}{\ln 7} + C$$

3) $$\int x\, 2^{(x^2)}\,dx$$ $u = x^2$, $du = 2x\,dx$ $$dx = \frac{du}{2x}$$

$$= \int \cancel{x}\, 2^{u} \frac{du}{2\cancel{x}} = \frac{1}{2}\frac{2^{u}}{\ln 2} = \frac{1}{2}\frac{2^{x^2}}{\ln 2} + C$$

(4) $\int 2^x e^{4x}\,dx$

$$2^x = e^{\ln 2^x} = e^{x\ln 2}$$

$$= \int e^{x\ln 2} e^{4x}\,dx = \int e^{x\ln 2 + 4x}\,dx$$

$$= \int e^{(\ln 2 + 4)x}\,dx$$

$u = (\ln 2 + 4)x$

$du = (\ln 2 + 4)\,dx$

$dx = \dfrac{du}{\ln 2 + 4}$

$$= \int e^u \frac{du}{\ln 2 + 4} = \frac{1}{\ln 2 + 4} e^u = \frac{1}{\ln 2 + 4} e^{(\ln 2 + 4)x} + C$$

RECALL

$$\frac{d}{dx}(\text{ARCSIN}\,x) = \frac{1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx}(\text{ARCTAN}\,x) = \frac{1}{1 + x^2}$$

$$\frac{d}{dx}(\text{ARCSEC}\,x) = \frac{1}{|x|\sqrt{x^2 - 1}}$$

THEREFORE:

$$\int \frac{1}{\sqrt{1 - x^2}}\,dx = \text{ARCSIN}\,x + C$$

$$\int \frac{1}{1+x^2}\,dx = \operatorname{ARCTAN} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \operatorname{ARCSEC} x + C$$

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① $\displaystyle\int \frac{1}{\sqrt{1-9x^2}}\,dx$

$9x^2 = u^2$

$u = 3x$

$du = 3dx \qquad dx = \frac{du}{3}$

$$= \int \frac{1}{\sqrt{1-u^2}}\,\frac{du}{3} = \frac{1}{3}\operatorname{ARCSIN}(u)$$

$$= \frac{1}{3}\operatorname{ARCSIN}(3x) + C$$

② $\displaystyle\int \frac{1}{\sqrt{9-x^2}}\,dx$

$$9 - x^2 = 9 - \frac{9x^2}{9} = 9\left(1 - \frac{x^2}{9}\right)$$

$$= 9\left(1 - \left(\frac{x}{3}\right)^2\right)$$

$$= \int \frac{1}{\sqrt{9\left(1-\left(\frac{x}{3}\right)^2\right)}}\,dx$$

$u = \frac{x}{3}$

$du = \frac{1}{3}dx \qquad dx = 3du$

$$= \int \frac{1}{\sqrt{9(1-u^2)}}\,3du = \int \frac{\cancel{3}}{\cancel{\sqrt{9}}\sqrt{1-u^2}}\,du$$

$$= \arcsin(u) = \arcsin\left(\frac{x}{3}\right) + C$$

③ $$\int \frac{1}{x^2+4}\,dx$$

$$x^2+4 = \frac{4x^2}{4} + 4 = 4\left(\frac{x^2}{4}+1\right)$$

$$= 4\left(\left(\frac{x}{2}\right)^2+1\right)$$

$$= \int \frac{1}{4\left(\left(\frac{x}{2}\right)^2+1\right)}\,dx = \frac{1}{4}\int \frac{1}{\left(\frac{x}{2}\right)^2+1}\,dx$$

$$u = \frac{x}{2} \qquad du = \frac{1}{2}\,dx \qquad dx = 2\,du$$

$$= \frac{1}{\cancel{4}\,2}\int \frac{1}{u^2+1}\,\cancel{2}\,du = \frac{1}{2}\arctan u$$

$$= \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$$

$$\int \frac{1}{x^2+a^2}\,dx$$

$$x^2+a^2 = \frac{a^2x^2}{a^2} + a^2 = a^2\left(\frac{x^2}{a^2}+1\right)$$

$$= a^2\left(\left(\frac{x}{a}\right)^2+1\right)$$

$$= \int \frac{1}{a^2\left(\left(\frac{x}{a}\right)^2+1\right)}\,dx = \frac{1}{a^2}\int \frac{1}{\left(\frac{x}{a}\right)^2+1}\,dx$$

$$u = \frac{x}{a} \qquad du = \frac{1}{a}\,dx \qquad dx = a\,du$$

$$= \frac{1}{a^{\cancel{2}}} \int \frac{1}{u^2+1} \cancel{a}\, du = \frac{1}{a} \text{ARCTAN } u$$

$$= \frac{1}{a} \text{ARCTAN}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{x^2+a^2}\, dx = \frac{1}{a} \text{ARCTAN}\left(\frac{x}{a}\right) + C$$

$$\int \frac{x}{1+x^4}\, dx \qquad u = x^2 \qquad du = 2x\, dx \qquad dx = \frac{du}{2x}$$

$$= \int \frac{\cancel{x}}{1+u^2} \frac{du}{2\cancel{x}} = \frac{1}{2} \int \frac{1}{1+u^2}\, du$$

$$= \frac{1}{2} \text{ARCTAN}(u) = \frac{1}{2} \text{ARCTAN}(x^2) + C$$

$$\int \frac{1}{x^2-4x+7}\, dx$$

$$x^2 - 4x + 4 + 7 - 4 = (x-2)^2 + 3$$

$$= \int \frac{1}{(x-2)^2+3}\, dx \qquad u = x-2 \qquad du = dx$$

$$= \int \frac{1}{u^2+3}\, du = \frac{1}{\sqrt{3}} \text{ARCTAN}\left(\frac{u}{\sqrt{3}}\right)$$

$$= \frac{1}{\sqrt{3}} \text{ARCTAN}\left(\frac{x-2}{\sqrt{3}}\right) + C$$

$$\int \frac{x+3}{\sqrt{9-x^2}}\,dx = \int \frac{x}{\sqrt{9-x^2}}\,dx + \int \frac{3}{\sqrt{9-x^2}}\,dx$$

1 2

1 $\displaystyle\int \frac{x}{\sqrt{9-x^2}}\,dx$

$u = 9 - x^2$

$du = -2x\,dx$

$dx = \dfrac{du}{-2x}$

$$= \int \frac{\cancel{x}}{\sqrt{u}}\,\frac{du}{-2\cancel{x}} = -\frac{1}{\cancel{2}}\,\cancel{2}\,u^{\frac{1}{2}} = -\sqrt{9-x^2}$$

2 $\displaystyle\int \frac{3}{\sqrt{9-x^2}}\,dx = 3\int \frac{1}{\sqrt{9-x^2}}\,dx$

$$= 3\,\text{ARCSIN}\left(\frac{x}{3}\right)$$

$$\int \frac{x+3}{\sqrt{9-x^2}}\,dx = -\sqrt{9-x^2} + 3\,\text{ARCSIN}\left(\frac{x}{3}\right) + C$$